

Connecting the little dots in polarized light

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ABSTRACT

Recent observations of the local Little Red Dot (LRD) analog SDSS J1025+1402 have revealed that the optical continuum is polarized at the $\sim 1.5\%$ level, while broad H α has lower polarization ($\sim 0.7\%$) and a different polarization angle. We explain these properties in a scenario in which LRDs are dust-obscured Little Blue Dots (LBDs) viewed at high inclination and powered by super-Eddington accretion. In this framework, electron scattering in the geometrically thick inner accretion flow produces intrinsic continuum polarization, while unpolarized emission from the outer thin disk dilutes the signal, particularly at optical wavelengths. A circumnuclear dust screen contributes dichroic polarization to both the continuum and broad-line emission. Broad H α lacks the intrinsic disk component and therefore has lower polarization and a different position angle. We show that this model quantitatively reproduces the observed polarization properties, although degeneracies between accretion rate and inclination remain. These results illustrate the potential of spectropolarimetry as a probe of accretion-flow geometry and orientation in the little-dot population, complementary to spectral-energy-distribution fitting and line diagnostics.

Key words. Accretion (14); Active galactic nuclei (16); James Webb Space Telescope (2291); Supermassive black holes (1663)

1. Introduction

The James Webb Space Telescope has revealed a substantial population of compact broad-line AGNs at redshifts $z \gtrsim 4$, powered by accretion onto black holes with inferred masses in the range $\sim 10^6\text{--}10^8 M_\odot$ (e.g., Harikane et al. 2023; Maiolino et al. 2024; Taylor et al. 2025; Juodžbalis et al. 2026). A subset of these objects displays red, V-shaped UV–optical continua and is commonly identified as Little Red Dots (LRDs), while the majority show bluer continua and are classified as Little Blue Dots (LBDs) (Brazzini et al. 2026; Geris et al. 2026). In current JWST samples, LRDs account for only $\sim 10\%\text{--}30\%$ of the compact BLAGN population, although the inferred fraction depends on luminosity, redshift, and selection band (Hainline et al. 2025; Taylor et al. 2025; Madau & Maiolino 2026b), suggesting that the observed diversity is more likely a statistical property of a single population than evidence for two fundamentally distinct classes (Billand et al. 2026).

In Madau & Maiolino (2026a), we proposed that LRDs and LBDs are different inclination-dependent manifestations of the same underlying population of compact, super-Eddington AGNs. In that framework, the central engine consists of a geometrically thick, radiation-supported accretion flow that emits anisotropically, surrounded by a broad-line region (BLR) concentrated toward the equatorial plane and a dusty circumnuclear component with a modest covering factor. Lines of sight at relatively low inclination avoid significant dust attenuation and appear as LBDs, whereas

LRDs correspond to the same systems viewed along directions that intercept the dusty component.

This orientation-based picture has recently received two additional lines of support. First, Madau & Maiolino (2026b) showed that the observed UV luminosity function of LRDs can be reproduced as the dust-intercepted minority tail of the compact BLAGN parent population, and that the same forward model also predicts the larger inferred black-hole masses of UV-selected LRDs relative to unobscured LBDs as a consequence of selection through dust attenuation. Second, Madau et al. (2026) demonstrated that a stratified BLR embedded in this same super-Eddington geometry can account for the extended, nearly exponential Balmer wings observed in both LRDs and LBDs, without invoking electron scattering as the dominant line-broadening mechanism. Additional consistency comes from AGN-powered narrow-line ratios and Ly α emission, which show that ionizing radiation escapes from LRD nuclei, and from broad-Balmer equivalent widths and line ratios that are consistent with the predictions of the dusty, anisotropic model (Madau & Maiolino 2026a; Geris et al. 2026; Sok et al. 2026). Together, these results suggest that continuum colors, number demographics, black-hole mass estimates, and broad-line profiles can all be understood within a single rapidly accreting unification framework.

A remaining untested prediction of this scenario concerns the polarization properties of the continuum-emitting accretion flow itself. Because the observed radiation originates from an optically thick, scattering-dominated, highly

aspherical surface, the escaping continuum should acquire a non-zero and strongly inclination-dependent linear polarization. In this paper we compute the frequency-dependent continuum polarization by integrating the Stokes parameters over the visible disk surface in the Thomson-scattering regime. We show that, when combined with a modest dichroic contribution, this geometry can account for the nearly gray continuum polarization of J1025+1402 at the level of $\simeq 1.5\%$, as well as the lower polarization of broad H α and its position-angle offset relative to the continuum.

2. Polarization model

We compute the polarization of the emergent continuum using the same geometrical and radiative framework adopted in Madau (2026) for the inclination-dependent SED of super-Eddington flows. The calculation uses the same geometrically thick-disk geometry, the same treatment of non-local self-irradiation within the funnel, and the same self-shadowing that suppresses the visibility of the hot inner regions at high inclination. The polarized signal is therefore derived from the same radiation field that shapes the continuum SED and the BLR illumination in Madau & Maiolino (2026a).

2.1. Geometry

The super-Eddington accretion flow is modeled as the composite thick+thin solution described in Madau (2026): a radiation-supported, non-Keplerian torus extending from the inner radius r_{in} to a transition radius r_{thick} , smoothly matched to a standard Keplerian thin disk extending to r_{out} . We define the dimensionless accretion rate as $\dot{m} \equiv M/\dot{M}_{\text{Edd}}$, where $\dot{M}_{\text{Edd}} \equiv 10L_{\text{Edd}}/c^2$. The funnel height $h(r)$ reaches a maximum at $r = r_{\text{max}}$ and decreases again toward the outer transition region. The emitting funnel surface is described in cylindrical coordinates by the height profile $h(r)$ with slope $h'(r) = dh/dr$, and the corresponding surface element and outward unit normal are

$$d\Sigma = r\sqrt{1+h'^2} dr d\phi, \quad \hat{\mathbf{n}} = \frac{(-h' \cos \phi, -h' \sin \phi, 1)}{\sqrt{1+h'^2}}. \quad (1)$$

For an observer at inclination i from the symmetry axis, the line-of-sight unit vector is $\hat{\mathbf{k}} = (\sin i, 0, \cos i)$ and $\mu(r, \phi; i) \equiv \hat{\mathbf{n}} \cdot \hat{\mathbf{k}}$. Only surface elements with $\mu > 0$ and not blocked by self-shadowing contribute to the observed flux, using the same visibility conditions as in Madau (2026).

2.2. Radiative transfer on the funnel wall

The funnel photosphere operates in the Thomson-scattering dominated regime, $\kappa_{\text{abs}} \ll \kappa_{\text{es}}$. At the high accretion rates and temperatures characteristic of super-Eddington flows, the gas is fully ionized and the photospheric opacity is set almost entirely by electron scattering, with free-free and bound-free contributions negligible in the UV/optical band (Madau 2026). As a result, the outgoing continuum acquires net linear polarization through the angular asymmetry of the final electron scatterings near the photosphere. In addition to the locally generated flux, each surface element receives radiation from other visible parts of the funnel. Because the wall atmosphere has single-scattering albedo close to unity at UV/optical wavelengths, this intercepted

flux is not thermally absorbed and re-emitted as blackbody radiation. Instead, it is redistributed by multiple electron scatterings within the local wall atmosphere before contributing to the outgoing radiation field. The appropriate description is therefore a self-consistent multiple-scattering problem on the funnel surface, not a single-scatter source term.

We discretize the axisymmetric funnel wall into radial rings indexed by k . Following Madau (2026), the total outgoing spectral flux associated with ring k satisfies

$$\mathcal{F}_{\nu,k} = \mathcal{F}_{\nu,k}^{\text{local}} + \frac{1}{\pi} \sum_j B_{kj} \mathcal{F}_{\nu,j}, \quad (2)$$

where $\mathcal{F}_{\nu,k}^{\text{local}}$ is the locally generated flux and the geometric kernel

$$B_{kj} = \int_0^{2\pi} \frac{[-\hat{\mathbf{n}}_k \cdot \hat{\mathbf{L}}]_+ [\hat{\mathbf{n}}_j \cdot \hat{\mathbf{L}}]_+}{|\mathbf{L}|^2} d\Sigma_j(\phi), \quad d\Sigma_j = r_j \sqrt{1+h_j'^2} \Delta r d\phi, \quad (3)$$

is the dimensionless geometric coupling factor, such that B_{kj}/π is the fraction of the flux emitted by ring j that is intercepted by ring k . Here $\mathbf{L}$ points from a surface element at azimuth ϕ on ring j to a reference element on ring k , and $[x]_+ \equiv \max(x, 0)$ restricts the integral to mutually visible surface elements. The linear system in Equation (2) is solved by LU decomposition, thereby summing all orders of internal wall-to-wall scattering.

For wall-to-wall coupling we adopt the Lambertian closure $I_\nu = \mathcal{F}_\nu/\pi$, so B_{kj} maps the scalar outgoing flux between rings and Equation (2) sums repeated re-irradiation. After solving for $\mathcal{F}_{\nu,k}$, we assign its angular distribution and polarization toward the observer using the Chandrasekhar solution for a semi-infinite conservative electron-scattering atmosphere. Thus internal coupling is treated as isotropic and scalar, whereas the final radiation is anisotropic and polarized. A fully direction-dependent polarized radiative-transfer calculation is left for future work.

With these prescriptions, the wall solve determines the total scalar outgoing flux $\mathcal{F}_\nu(r)$, including both locally generated radiation and energy returned by other parts of the funnel. The angular dependence of the radiation reaching the observer is then written as

$$I_\nu(r, \mu) = \frac{\mathcal{F}_\nu(r)}{2\pi} \mathcal{H}(\mu), \quad (4)$$

$$Q_\nu(r, \mu) = \frac{\mathcal{F}_\nu(r)}{2\pi} p(\mu) \mathcal{H}(\mu), \quad (5)$$

where $\mathcal{H}(\mu) \simeq 0.85 + 1.725\mu$ is the limb-darkening function, normalized so that $\int_0^1 \mathcal{H}(\mu) \mu d\mu = 1$,¹ and $p(\mu)$ is the Chandrasekhar polarization fraction. The latter vanishes for normal emergence ($\mu = 1$) and rises to 11.7% at grazing emergence ($\mu \rightarrow 0$). Both functions are evaluated at the observer angle $\mu = \mu(r, \phi; i)$.

We neglect circular polarization, as appropriate for cold-electron Thomson scattering without incident circular polarization or magnetic propagation effects. We place the

¹ Our calculations implement the exact Chandrasekhar $\mathcal{H}$ -function solution; the approximate expression for $\mathcal{H}(\mu)$ is given here only for illustration.

observer in the xz -plane and use the projected disk rotation axis as the reference direction for Stokes Q_ν . Reflection symmetry about this plane pairs elements at (r, ϕ) and $(r, -\phi)$ with equal Stokes Q_ν contributions and opposite Stokes U_ν contributions, where the axes defining U_ν are rotated by 45° relative to those defining Q_ν . The U_ν contributions therefore cancel pairwise in the azimuthal integration, leaving the net Stokes vector purely Q_ν in the symmetry-aligned frame. Rotation to the celestial reference frame then generally produces nonzero observed Q_ν and U_ν , without breaking the underlying axial symmetry. This cancellation is illustrated with a conical-funnel model in Appendix A.

2.3. Observed Stokes parameters

The observed Stokes fluxes are obtained by integrating the specific intensities from Equations (4)–(5) over the visible photospheric surface of the geometrically thick flow. For each surface element at (r, ϕ) , the local linear-polarization direction is perpendicular to the plane defined by the surface normal and the observer's line of sight. We denote its position angle relative to the projected disk rotation axis by $\chi(r, \phi; i)$. If ψ_{axis} is the position angle of that axis relative to the fixed celestial Stokes- Q direction, the corresponding polarization angle in the observer's reference frame is

$$\chi_{\text{sky}}(r, \phi; i) = \chi(r, \phi; i) + \psi_{\text{axis}}. \quad (6)$$

The axisymmetric calculation does not determine ψ_{axis} . In the fits below, we instead use the celestial polarization position angle of the integrated disk component, ψ_{disk} , directly as a nuisance parameter.

The projected area element is $\mu d\Sigma$, and the integration extends over all surface elements with $\mu > 0$ that are not blocked by self-shadowing. The celestial-frame Stokes fluxes are then

$$f_{\nu, \text{obs}} = \frac{1}{D^2} \int_{\Sigma_{\text{vis}}} I_\nu(r, \mu) \mu d\Sigma, \quad (7)$$

$$Q_{\nu, \text{sky}} = \frac{1}{D^2} \int_{\Sigma_{\text{vis}}} Q_\nu(r, \mu) \cos 2\chi_{\text{sky}} \mu d\Sigma, \quad (8)$$

$$U_{\nu, \text{sky}} = \frac{1}{D^2} \int_{\Sigma_{\text{vis}}} Q_\nu(r, \mu) \sin 2\chi_{\text{sky}} \mu d\Sigma. \quad (9)$$

Here D is the distance to the source. The quantity $Q_\nu(r, \mu)$ inside the integrals is the local polarized specific intensity defined by Equation (5), whereas $Q_{\nu, \text{sky}}$ and $U_{\nu, \text{sky}}$ are the surface-integrated Stokes fluxes. Similarly, $\mathcal{F}_\nu(r)$ is the local outgoing surface flux determined by Equation (2), whereas $f_{\nu, \text{obs}}$ is the corresponding surface-integrated observed flux.

2.4. Polarization dilution by the outer thin disk

The outer standard thin disk is included in the calculation but treated as unpolarized, contributing only flux that dilutes the polarized thick-flow signal. The total observed flux density is therefore

$$f_{\nu, \text{obs}}^{\text{tot}} = f_{\nu, \text{obs}} + f_{\nu, \text{obs}}^{\text{thin}}, \quad (10)$$

whereas $Q_{\nu, \text{sky}}$ and $U_{\nu, \text{sky}}$ receive no thin-disk contribution. The polarization fraction predicted by the accretion-disk

model is consequently

$$p_{\text{disk}}(\nu) = \frac{\sqrt{Q_{\nu, \text{sky}}^2 + U_{\nu, \text{sky}}^2}}{f_{\nu, \text{obs}} + f_{\nu, \text{obs}}^{\text{thin}}}. \quad (11)$$

Because the polarized amplitude is invariant under rotations of the Stokes reference frame, the same polarization fraction is obtained in the symmetry-aligned frame, where the surface-integrated Stokes U_ν vanishes.

The assumption of negligible thin-disk polarization is motivated by the properties of its atmosphere. In the cooler Keplerian regime, absorption opacity becomes important and the classical Chandrasekhar (1960) solution for a pure electron-scattering atmosphere no longer applies. Absorption can either suppress or enhance the polarization, depending on the thermal source-function gradient, whereas magnetic Faraday rotation can reduce it further (Laor et al. 1990; Koratkar & Blas 1999; Agol et al. 1998; Taverna et al. 2021). Detailed atmosphere calculations generally predict weak, model-dependent optical/UV polarization, typically at or below the percent level. We therefore neglect the intrinsic thin-disk Stokes fluxes while retaining its continuum flux. This approximation is motivated by atmospheric physics, not by axisymmetry, since an inclined axisymmetric scattering disk can produce finite linear polarization.

The unpolarized thin-disk flux lowers the net polarization fraction, particularly at optical wavelengths where its relative contribution is larger. The resulting wavelength-dependent dilution produces a mild UV-to-optical decline in $p_{\text{disk}}(\lambda)$. Figure 1 illustrates this effect for a fiducial super-Eddington accretor with $M_{\text{BH}} = 10^{7.5} M_\odot$ and $\dot{m} = 12.4$. The polarized thick-flow emission is divided into the inner funnel ($r_{\text{in}} \leq r \leq r_{\text{max}}$) and the outer rim ($r_{\text{max}} < r \leq r_{\text{thick}}$); the standard thin disk ($r_{\text{thick}} < r \leq r_{\text{out}}$) contributes only diluting flux. At $i \lesssim 60^\circ$, azimuthal cancellation keeps the net polarization below $\sim 1\%$. Toward edge-on orientations, self-shadowing suppresses the inner funnel and increases the relative importance of the outer rim. At $i = 80^\circ$, the polarization reaches 1.5% at 1500 Å and 1.0% at 6500 Å; the difference reflects the increasing thin-disk contribution toward longer wavelengths.

2.5. Dependence on accretion rate and wavelength

The dependence of p_{disk} on $\dot{m}$ is shown in Figure 2. The polarization generally rises with inclination, apart from small non-monotonic features at low inclination where azimuthal cancellation is nearly complete. At $i = 85^\circ$, $p_{\text{disk}}(1500 \text{ Å})$ reaches 4.2%, 3.6%, 1.9%, and 1.3% for $\dot{m} = 78.8, 31.7, 12.4$, and 7.1, respectively. The dependence on $\dot{m}$ is governed primarily by the extent of the unpolarized outer thin disk rather than by the geometry of the geometrically thick flow alone. At higher accretion rates, r_{thick} is larger and the thin disk contributes less of the total optical flux, leaving the thick-flow polarization less diluted. At lower accretion rates, r_{thick} decreases and the thin disk extends over a wider radial range, so its unpolarized flux increasingly dilutes the optical polarization. Consequently, p_{disk} increases with $\dot{m}$ at moderate and high inclinations.

The wavelength dependence also varies with $\dot{m}$. Comparing the two panels of Figure 2, at $i = 85^\circ$ we find

$$\frac{p_{\text{disk}}(1500 \text{ Å})}{p_{\text{disk}}(6500 \text{ Å})} = 1.08, 1.14, 1.56, \text{ and } 1.74 \quad (12)$$

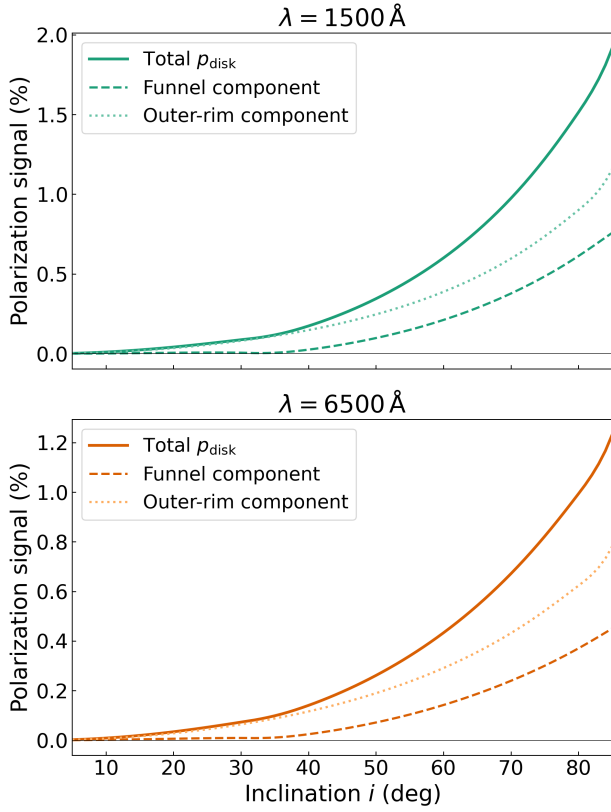


Fig. 1. Predicted continuum polarization versus inclination for $M_{\text{BH}} = 10^{7.5} M_{\odot}$ and $\dot{m} = 12.4$ at rest-frame 1500 Å (top) and 6500 Å (bottom). Solid curves show p_{disk} ; dashed and dotted curves show the signed inner-funnel and outer-rim contributions to $Q_{\nu, \text{sky}}/f_{\nu, \text{obs}}^{\text{tot}}$. The unpolarized thin disk dilutes the signal, especially at longer wavelengths.

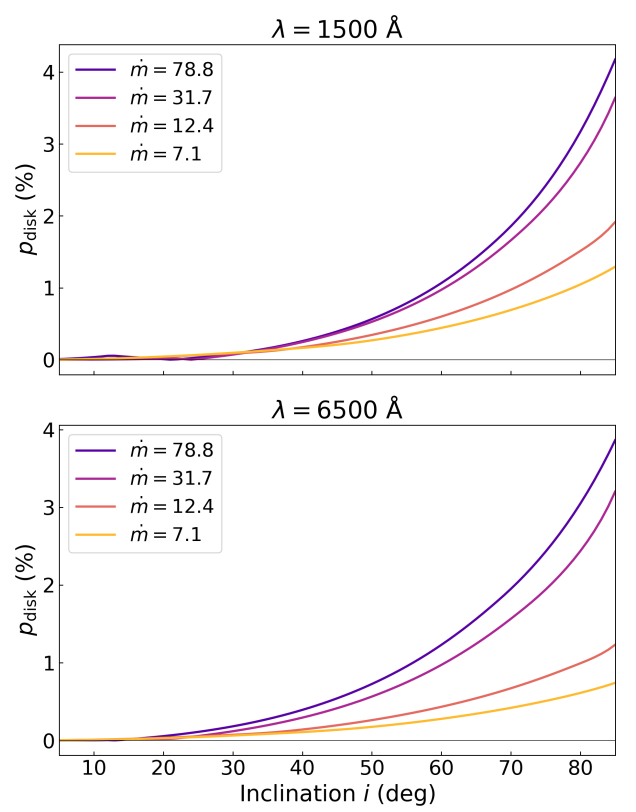


Fig. 2. Net disk polarization versus inclination for $M_{\text{BH}} = 10^{7.5} M_{\odot}$ and several accretion rates, at rest-frame 1500 Å (top) and 6500 Å (bottom). Polarization generally increases with inclination and $\dot{m}$; thin-disk dilution makes the UV-to-optical decline stronger at lower $\dot{m}$.

for $\dot{m} = 78.8, 31.7, 12.4$, and 7.1 , respectively. Thus the two higher-rate models are nearly gray, whereas the lower-rate models show a progressively stronger UV-to-optical decline owing to thin-disk dilution. Figure 3 shows the full wavelength dependence for the fiducial $\dot{m} = 12.4$ model.

3. Case study: the LRD analog SDSS J1025+1402

Recent optical spectropolarimetry of the nearby LRD analog SDSS J1025+1402 provides direct constraints on the continuum-emitting region of an LRD-like source (D'Eugenio et al. 2026). Located at $z \simeq 0.1$, J1025+1402 shares the defining properties of high-redshift LRDs, including a compact morphology, a V-shaped UV–optical spectral energy distribution, broad Balmer emission lines with extended non-Gaussian wings, a red optical continuum, and extreme X-ray weakness (Lin et al. 2026; Ji et al. 2026). Its proximity enables signal-to-noise levels currently inaccessible for the high-redshift population.

Using VLT/FORS2 optical spectropolarimetry, D'Eugenio et al. (2026) measured a nearly gray continuum polarization of $p_{\text{cont}} = (1.53 \pm 0.04_{\text{rand}} \pm 0.20_{\text{syst}})\%$ over the rest-frame wavelength range 5300–6650 Å. After continuum subtraction, broad H α is less polarized, with $p_{\text{H}\alpha} = 0.58\text{--}0.84\%$, depending on the estimator. Its polarization angle differs significantly from that of the

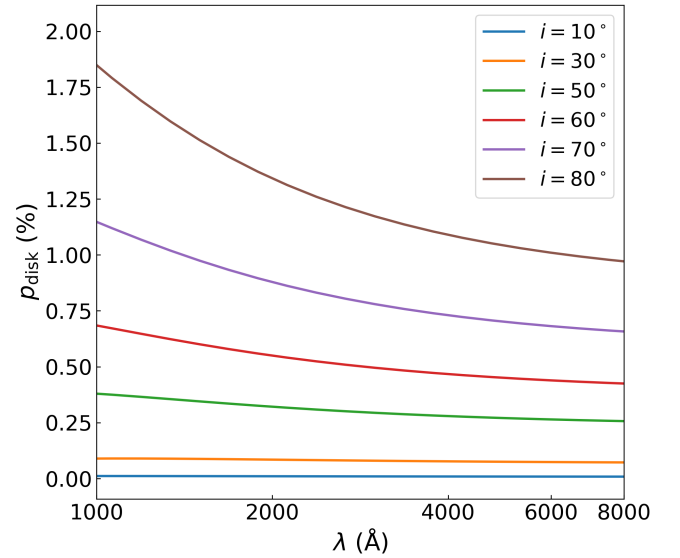


Fig. 3. Net disk polarization spectrum for $M_{\text{BH}} = 10^{7.5} M_{\odot}$, $\dot{m} = 12.4$, and several inclinations. The polarization increases with inclination and declines toward longer wavelengths as the unpolarized thin disk contributes a larger fraction of the flux.

continuum, with

$$\Delta\vartheta \equiv \vartheta_{\text{H}\alpha} - \vartheta_{\text{cont}} = -48^\circ \pm 4^\circ. \quad (13)$$

This position-angle offset provides the principal constraint on the two-component polarization model developed below.

3.1. Model

We model the normalized continuum Stokes vector as the sum of two contributions: intrinsic scattering polarization from the super-Eddington flow and dichroic polarization from aligned dust grains in a dusty screen along the LRD sightline. We associate this screen with the circumnuclear dusty component of the LRD–LBD unification model, although the polarization calculation applies equally to any intervening dust-bearing structure along the line of sight. We define the normalized linear Stokes parameters as

$$q(\lambda) \equiv \frac{Q(\lambda)}{I(\lambda)}, \quad u(\lambda) \equiv \frac{U(\lambda)}{I(\lambda)}, \quad (14)$$

so that $p = (q^2 + u^2)^{1/2}$. A component with polarization fraction p and celestial position angle ψ contributes $q = p \cos 2\psi$ and $u = p \sin 2\psi$. All position angles are understood modulo 180° . At continuum wavelengths,

$$q_{\text{cont}}(\lambda) = p_{\text{disk}}(\lambda) \cos 2\psi_{\text{disk}} + p_{\text{dic}}(\lambda) \cos 2\psi_{\text{dic}}, \quad (15)$$

$$u_{\text{cont}}(\lambda) = p_{\text{disk}}(\lambda) \sin 2\psi_{\text{disk}} + p_{\text{dic}}(\lambda) \sin 2\psi_{\text{dic}}, \quad (16)$$

where $p_{\text{disk}}(\lambda)$ is the polarization fraction predicted by the disk model, including dilution by the unpolarized outer thin disk, at inclination i and accretion rate $\dot{m}$. The quantities $p_{\text{dic}}(\lambda)$ and ψ_{dic} describe the dichroic component, while ψ_{disk} is the celestial polarization position angle of the disk component. Equations (15) and (16) use the first-order, small-polarization limit and neglect terms of order $p_{\text{disk}}p_{\text{dic}}$.

We assume that the extended broad-line emission carries negligible intrinsic or scattered polarization and passes through the same dichroic screen as the continuum. Broad H α therefore carries only the dichroic Stokes vector.

If $f_{\text{cont}}(\lambda) \equiv I_{\text{cont}}(\lambda)/I_{\text{obs}}(\lambda)$ is the continuum fraction measured from the observed intensity spectrum, the model across H α becomes

$$q_{\text{mod}}(\lambda) = q_{\text{dic}}(\lambda) + f_{\text{cont}}(\lambda)q_{\text{disk}}(\lambda), \quad (17)$$

$$u_{\text{mod}}(\lambda) = u_{\text{dic}}(\lambda) + f_{\text{cont}}(\lambda)u_{\text{disk}}(\lambda). \quad (18)$$

Outside the line window, $f_{\text{cont}} = 1$, and these expressions reduce to the continuum model above. The retained bins across the broad-line profile therefore constrain the relative amplitudes and orientations of the disk and dichroic components; the continuum-subtracted H α polarization fraction and position angle are not included as separate summary constraints. This baseline fit uses the observed total-intensity spectrum $I_{\text{obs}}(\lambda)$ and its fitted continuum $I_{\text{cont}}(\lambda)$ to determine $f_{\text{cont}}(\lambda)$ and hence the line dilution. A fully forward model could instead predict the H α luminosity and profile through a photoionization calculation. Any additional scalar foreground attenuation multiplies all Stokes fluxes by the same wavelength-dependent factor and therefore cancels from the normalized Stokes parameters. This is a transmission geometry: dust scattering would instead require off-axis photons to be redirected into the line of sight and is not included in the broad-line model.

The disk polarization $p_{\text{disk}}(\lambda)$ is nearly gray across the limited optical wavelength range of the observations. Although electron scattering itself is wavelength-independent, the disk-integrated polarization retains a weak wavelength

dependence through the radial weighting of the emission and dilution by the outer thin disk. For the best-fitting disk model discussed below, p_{disk} changes by only $\simeq 0.01$ percentage points between 5300 and 6650 Å. Any appreciable residual slope in the modeled continuum polarization is therefore governed primarily by the dichroic component, for which we adopt the Serkowski law (Serkowski et al. 1975),

$$p_{\text{dic}}(\lambda) = p_{\text{max}} \exp \left[-K \ln^2 \left(\frac{\lambda_{\text{max}}}{\lambda} \right) \right], \quad (19)$$

where the amplitude p_{max} is fitted directly. We fix $\lambda_{\text{max}} = 5500$ Å, the canonical value for diffuse Galactic sightlines (Serkowski et al. 1975), and set $K = 0.92$ using the empirical relation $K \simeq 1.66(\lambda_{\text{max}}/\mu\text{m}) + 0.01$ (Whittet et al. 1992).

We verified that the results are insensitive to the adopted dichroic wavelength dependence. Varying λ_{max} from 4000 to 7000 Å, with K tied through the Whittet relation, changes the fitted p_{max} by less than 20% and the minimum χ^2 by less than 0.5, while leaving the inferred disk polarization angle essentially unchanged. This insensitivity arises because the Serkowski curve is nearly achromatic across the narrow optical baseline, so its shape cannot be constrained by the present data.

3.2. Fit and results

In our baseline fit, we do not decompose H α into broad and narrow components. The complete observed line profile is assumed to pass through the same dichroic screen. Residual structure near line center may therefore reflect an unpolarized narrow-line contribution, or absorption-related effects, neither of which is included in our model (but see D'Eugenio et al. 2026). The FITS data provided by D'Eugenio et al. contain adaptively binned I , p , and polarization position angle ϑ , together with their per-bin uncertainties. We reconstruct the normalized Stokes parameters as

$$q = p \cos 2\vartheta, \quad u = p \sin 2\vartheta. \quad (20)$$

For each wavelength bin, we propagate the reported uncertainties in (p, ϑ) through this transformation, using a linearized Gaussian approximation with ϑ in radians, to obtain a 2×2 covariance matrix C_k for (q_k, u_k) . The corresponding chi-square statistic is

$$\chi^2 = \sum_k \Delta \mathbf{s}_k^T C_k^{-1} \Delta \mathbf{s}_k, \quad \Delta \mathbf{s}_k \equiv \mathbf{s}_k - \mathbf{s}_{k,\text{mod}}, \quad \mathbf{s}_k = (q_k, u_k). \quad (21)$$

This procedure retains the covariance between q and u within each wavelength bin. Correlations between different adaptive wavelength bins are negligible, and the corresponding covariance blocks are set to zero.

After excluding the telluric B band and selected line-contaminated regions, we retain 69 of the supplied adaptive bins: 40 line-free continuum bins and 29 bins spanning the H α window. An inverse-variance-weighted linear fit to the supplied I values in the 40 continuum bins defines $I_{\text{cont}}(\lambda)$ and hence $f_{\text{cont}}(\lambda)$. The likelihood therefore contains 138 normalized-Stokes measurements. At each $(\dot{m}, i)$ point in the precomputed thick-disk grid, we minimize over the three continuous parameters p_{max} , ψ_{disk} , and ψ_{dic} . Counting also

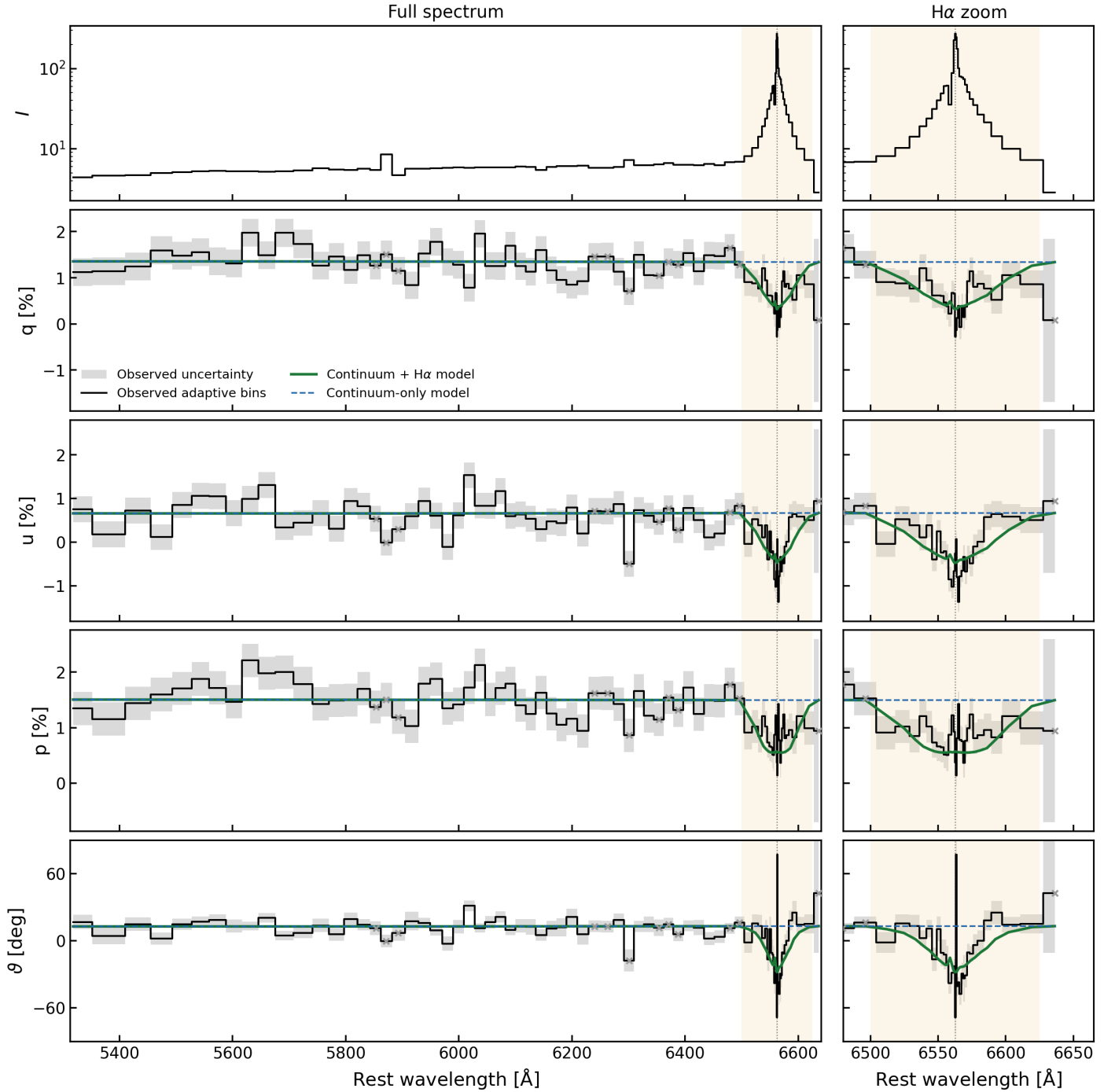


Fig. 4. Full-spectrum fit to SDSS J1025+1402 over the fitted range (left) and around broad H α (right). Rows show the intensity used to determine f_{cont} , normalized Stokes q and u , the derived p , and the polarization position angle θ . Black steps and gray bands are the FORS2 data and 1σ uncertainties; green curves show the full model and dashed blue curves the continuum-only prediction. Shading marks the fitted line window, and gray crosses mark the masked bins. The model predicts a smooth position-angle change as the disk-polarized continuum is diluted relative to the dichroically polarized line. The sharp central excursion occurs only in the data, where p is small and the position angle is poorly constrained. Only retained q and u values enter the likelihood.

the two scanned physical parameters gives 133 nominal degrees of freedom. No continuum intercept, Stokes slope, continuum polarization, or continuum-subtracted H α summary statistic is included separately in the likelihood, because all of these quantities are derived from the same fitted spectrum. The best-fit model is shown in Figure 4, and the principal results are summarized in Table 1. The formal minimum occurs at $\dot{m} = 31.7$ and $i = 70^\circ$, with $\chi^2 = 208$ and a global nominal $\chi^2/\text{dof} = 1.56$. However, the fit exhibits

an almost complete degeneracy between accretion rate and inclination. A model with $\dot{m} = 78.8$ and $i = 65^\circ$ differs by only $\Delta\chi^2 = 0.3$, while the optimized values of p_{max} , ψ_{disk} , and ψ_{dic} are essentially unchanged. The polarization data therefore constrain primarily the continuum polarization amplitude rather than $\dot{m}$ and i separately.

The decomposition of the best-fitting Stokes vectors is shown in Figure 5, while Table 1 lists the fitted parameters and derived polarization components. The disk and

dichroic vectors point in different directions and partially cancel. For plausible extinction parameters, $A_V \simeq 2\text{--}4$ mag and $R_V \simeq 3\text{--}4$, the fitted dichroic amplitude corresponds to $p_{\max}/E(B-V) \simeq 0.4\text{--}1.2\%$ mag $^{-1}$, or approximately 5–13% of the classical Galactic upper envelope. This low effective efficiency need not imply weak grain alignment. Galactic sightlines show that p/A_V can decline above $A_V \sim 1$ mag because polarization from dust-bearing regions with different magnetic-field orientations partially cancels (Bijias et al. 2022).

As a diagnostic, we fit the total and polarized-flux continua as $I_\lambda \propto \lambda^{\beta_I}$ and $pI_\lambda \propto \lambda^{\beta_{p \times I}}$, respectively. The model predicts $\Delta\beta \equiv \beta_{p \times I} - \beta_I = -0.03$, compared with the observed value of -0.80 ± 0.57 , a difference of only 1.35σ . Direct linear fits to the continuum bins yield slopes of $-0.186 \pm 0.150\%$ and $-0.184 \pm 0.149\%$ per 1000 Å in q and u , respectively, whereas the corresponding model slopes are -0.012% and $+0.005\%$ per 1000 Å. These differences are likewise only $1.2\text{--}1.3\sigma$. The current data therefore do not reveal a statistically significant wavelength-dependent departure from the nearly gray model. Because $\Delta\beta$ and the fitted continuum slopes are derived from the same spectral bins used in the fit, they are reported only as diagnostics and are not included as additional terms in Equation (21).

The directly measured polarization-angle offset between the continuum and the broad H α core is $\Delta\vartheta = -48^\circ \pm 4^\circ$ (stat) $\pm 12^\circ$ (epoch systematic) (D'Eugenio et al. 2026). The model offset is -43° , consistent with the measurement once the epoch-to-epoch systematic uncertainty is included. It arises because the continuum angle is set by the vector sum of the intrinsic disk and dichroic components, whereas the broad-line angle reflects the dichroic component alone. The latter is set by the mean projected magnetic-field orientation in the circumnuclear dusty screen, which need not coincide with the projected disk axis. No geometric asymmetry of the accretion flow is therefore required to explain the observed position-angle offset.

3.3. The H α core: a polarized narrow component?

The fit of Section 3.2 assumes that the entire H α profile is transmitted through the same dichroic screen, and it leaves systematic residuals within a few hundred km s $^{-1}$ of line center. The spatially extended narrow-line region supplies most of the intensity at the peak, so its polarization need not follow the nuclear continuum. We therefore extend the model with a polarized narrow component and refit the same 69 bins.

We continue to neglect intrinsic or scattered BLR polarization. The continuum carries the disk and dichroic components, while broad H α carries only the dichroic component. The narrow H α and [N II] emission is assumed to originate outside the dichroic screen and to be intrinsically unpolarized. A fraction of these photons may nevertheless acquire polarization by scattering into the line of sight. Let $f_{\text{NL}}(\lambda)$ be the fraction of the observed intensity contributed by the unscattered narrow-line emission. The extended model is

$$\begin{aligned} q_{\text{mod}}(\lambda) &= [1 - f_{\text{NL}}(\lambda)]q_{\text{dic}}(\lambda) + f_{\text{cont}}(\lambda)q_{\text{disk}}(\lambda) \\ &\quad + a_q\Phi(\lambda)f_{\text{cont}}(\lambda), \\ u_{\text{mod}}(\lambda) &= [1 - f_{\text{NL}}(\lambda)]u_{\text{dic}}(\lambda) + f_{\text{cont}}(\lambda)u_{\text{disk}}(\lambda) \\ &\quad + a_u\Phi(\lambda)f_{\text{cont}}(\lambda), \end{aligned} \quad (22)$$

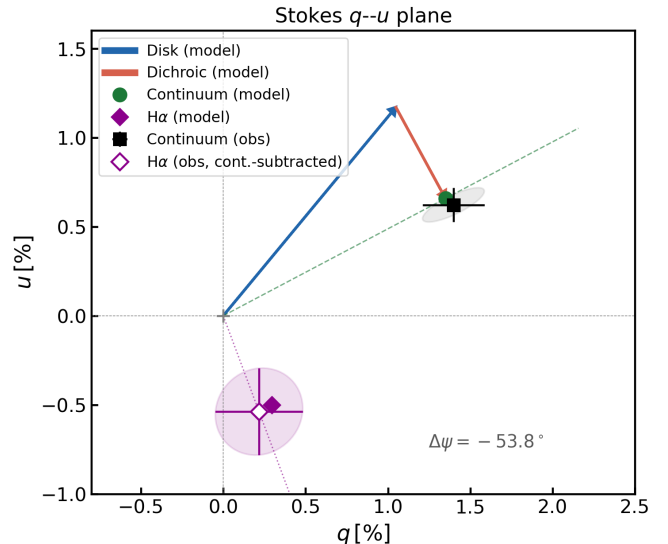


Fig. 5. Best-fit model in the Stokes q - u plane. Blue and red arrows show the intrinsic disk and dichroic vectors; their sum gives the continuum prediction (green circle), compared with the measurement (black square). The filled and open magenta diamonds show the model dichroic polarization at H α and the published continuum-subtracted measurement, respectively; the latter is diagnostic only. Ellipses and error bars show 1σ uncertainties, and the lines mark the corresponding position angles.

Here a_q and a_u are polarized Stokes-flux amplitudes normalized to the adjacent continuum intensity. Their position angle satisfies $\tan 2\psi_{\text{sc}} = a_u/a_q$, with the quadrant fixed by the signs of a_q and a_u . The template Φ combines narrow H α and [N II], with relative strengths tied to the total-intensity decomposition. Each seed line is convolved with a symmetric two-sided exponential of velocity scale v_e ; the instrumental line-spread function and adaptive spectral bins are included in the forward model. The intrinsic H α seed has FWHM = 92 km s $^{-1}$ (Ji et al. 2026).

Refitting $p_{\max}$, ψ_{disk} , and ψ_{dic} jointly with a_q , a_u , and v_e at the same grid point ($\dot{m} = 31.7$, $i = 70^\circ$) gives the values in Table 2. The chi-square decreases by 28, from 208 to 179, for three additional parameters; $\Delta\text{AIC} = -22$ and $\Delta\text{BIC} = -13$ for $n = 138$ data values. An otherwise identical fit with unpolarized direct narrow lines but no scattered component gives $\chi^2 = 240$; adding scattering lowers this by 61. These criteria favor the narrow-scattering component. The continuum parameters shift only modestly. At $T_e \sim 10^4$ K, the characteristic electron thermal speed is $\simeq 550$ km s $^{-1}$, about six times v_e , so thermal electron motions would smear the scattered profile too broadly. Grain thermal speeds are negligible, and the much larger optical opacity of dust in normally dusty gas further favors dust scattering (Capetti et al. 2021). Dust is inferred to be present in at least part of the NLR in many AGNs (Netzer & Laor 1993; Wills et al. 1993), providing a plausible scattering medium. The grains need not lie outside the NLR as a whole, but may be mixed with or surround the compact inner narrow-line-emitting region. The fitted $\psi_{\text{sc}} = -43^\circ$ nominally differs from both ψ_{disk} and ψ_{dic} , suggesting an additional polarizing structure, plausibly a polar scattering cone (Antonucci & Miller 1985). The observed core angle, $-34.0^\circ \pm 13.3^\circ$, lies between ψ_{sc} and ψ_{dic} , as expected for

Table 1. Direct full-spectrum polarization fit to SDSS J1025+1402.

Quantity	Value	Notes
<i>Data and likelihood</i>		
Retained bins	69	40 continuum + 29 H α
Fitted data values	138	q_k and u_k in every retained bin
χ^2	208	continuum: 93; H α : 115
dof	133	three continuous + two grid parameters
nominal χ^2/dof	1.56	208/133
<i>Best grid point and fitted parameters</i>		
$\dot{m}$	31.7	grid minimum
i	70°	grid minimum
p_{max}	$0.60 \pm 0.07\%$	formal local error
ψ_{disk}	$24^\circ \pm 2^\circ$	formal local error
ψ_{dic}	$-30^\circ \pm 2^\circ$	formal local error
<i>Derived diagnostics</i>		
p_{disk}	1.6%	mean intrinsic disk polarization
p_{cont}	1.5%	mean continuum model
ϑ_{cont}	13°	mean continuum model
$p_{\text{dic, H}\alpha}$	0.6%	line polarization in the model
$\psi_{\text{dic}} - \psi_{\text{disk}}$	-54°	component-angle difference
$p_{\text{max}}/E(B - V)$	0.4–1.2% mag ⁻¹	for $A_V = 2\text{--}4$ mag and $R_V = 3\text{--}4$
$\Delta\beta$	-0.03	observed: -0.80 ± 0.57 ; not fitted

Notes. The likelihood contains the 69 retained (q_k, u_k) pairs rather than continuum or H α summary statistics. The quoted uncertainties on the three continuous parameters are local-curvature estimates scaled by the reduced χ^2 . The Serkowski parameters $\lambda_{\text{max}} = 5500$ Å and $K = 0.92$ are fixed.

a blend of scattered narrow-line and dichroically polarized broad-line flux.

The measured $f_{\text{sc}}p_{\text{sc}} = 0.9\%$ is an observed polarized-flux fraction. The fit also treats Balmer absorption as unpolarized and fixes the seed width from the total-intensity decomposition, because the polarimetric data cannot constrain it independently.

4. Discussion and Summary

The polarization predicted by a radiation-supported super-Eddington flow differs from that of a standard thin disk. Azimuthal cancellation suppresses the disk-integrated signal at low inclination, while projection and self-shadowing allow it to reach several percent toward edge-on views. The polarization also increases with $\dot{m}$, because the unpolarized outer thin disk contributes less dilution as r_{thick} expands. High- $\dot{m}$ models are consequently nearly gray, whereas lower-rate models show a larger UV-to-optical decline.

Applied to the spectropolarimetric data of SDSS J1025+1402, the model favors a high-accretion-rate, high-inclination geometry but leaves a strong $\dot{m}$ - i degeneracy. The observed continuum is described by the vector sum of intrinsic disk polarization and dichroic polarization from the circumnuclear dusty component invoked in the LRD-LBD unification model. Broad H α is assumed to be intrinsically unpolarized and to pass through the same screen, so it retains only the dichroic component, whose position angle is set by the mean projected magnetic-field orientation in the dusty screen. The continuum-to-line offset therefore reflects the different orientations of the disk and dichroic Stokes vectors and does not require a non-axisymmetric disk. This transmission geometry is also

consistent with the absence of a position-angle swing across broad H α , because the screen applies one orientation to all line velocities.

Residuals at the H α + [N II] core motivate an additional component. If the direct narrow lines originate outside the dichroic screen and are unpolarized, allowing some of their photons to scatter into the line of sight decreases χ^2 by 28 for three added parameters relative to the baseline ($\Delta\text{BIC} = -13$), and by 61 relative to the corresponding unpolarized-narrow, no-scatter fit. The nominal scattering angle suggests a polar structure, while the narrow velocity scale favors dust over thermal electrons. This interpretation remains provisional: it fixes the seed-line decomposition and treats Balmer absorption as unpolarized.

The distinction between intrinsic disk polarization and an external scattering origin is relevant to comparisons with ordinary quasars. In the VLT/FORS2 sample of Capetti et al. (2021), the continuum and integrated broad H α generally have similar position angles, and swings across the line are common. Those properties are naturally produced when one equatorial scatterer redirects both continuum and BLR photons. J1025+1402 instead shows a large continuum-to-line offset and no broad-line swing (D'Eugenio et al. 2026). A single equatorial scatterer does not naturally produce both features, whereas a disk-polarized continuum viewed through a dichroic screen does. The modest optical polarization of most radio-quiet Narrow-Line Seyfert 1 galaxies may likewise reflect their predominantly lower inclinations; individual objects nevertheless show blue-rising polarization and line-dependent Stokes structure consistent with orientation playing an important role (Goodrich 1989; Leighly et al. 1997; Smith et al. 2004).

The polarization of J1025+1402 does not require an external continuum-scattering component: its observed ampli-

Table 2. Polarized narrow-component fit to the H α + [N II] core.

Quantity	Value	Notes
<i>Model setup</i>		
$\dot{m}$	31.7	inherited from Table 1
i	70°	inherited from Table 1
Seed FWHM	92 km s ⁻¹	fixed ^a
<i>Continuum: intrinsic disk polarization</i>		
ψ_{disk}	23° ± 2°	formal local error
<i>Dichroic screen: acts on continuum and broad Hα</i>		
p_{max}	0.5 ± 0.1%	formal local error
ψ_{dic}	-28° ± 3°	formal local error
$\psi_{\text{dic}} - \psi_{\text{disk}}$	-50°	component-angle difference
<i>Narrow Hα and [N II]: direct component</i>		
Direct narrow H α	unpolarized	93% of peak intensity
Direct [N II]	unpolarized	6% of peak intensity
<i>Narrow Hα and [N II]: scattered component</i>		
a_q	2 ± 3%	of adjacent-continuum Stokes flux
a_u	-19 ± 4%	of adjacent-continuum Stokes flux
ψ_{sc}	-43° ± 4°	derived formal error
$f_{\text{sc}} p_{\text{sc}}$	0.93%	observed polarized-flux fraction
v_e	85 ± 30 km s ⁻¹	formal local error
$\psi_{\text{sc}} - \psi_{\text{dic}}$	-15° ± 6°	derived formal error
<i>Goodness of fit and model comparison</i>		
χ^2	179	baseline on same bins: 208
nominal χ^2/dof	1.38	179/130; eight model parameters
$\Delta\chi^2$	-28	for three added parameters
ΔAIC	-22	relative to Table 1
ΔBIC	-13	relative to Table 1
No-scatter χ^2	240	same unpolarized direct narrow lines
χ^2 reduction vs. no scatter	61	relative to the no-scatter fit
<i>Not fitted</i>		
$\vartheta_{\text{H}\alpha, \text{core}}$	-34.0° ± 13.3°	observed ^b

Notes. The fit uses the same data and likelihood as Table 1. The disk grid point is fixed, while the three continuum and three scattering parameters are refitted jointly. The quoted uncertainties on fitted parameters are local-curvature estimates; uncertainties on derived quantities include correlations among the fitted parameters. ^(a) Seed width from Ji et al. (2026). ^(b) Observed value from D'Eugenio et al. (2026).

tude is reproduced by the disk-plus-dichroic model. More generally, continuum polarization substantially exceeding the few-percent levels predicted by the disk-plus-dichroic model could signal an additional contribution from polar scattering. Continuum radiation preferentially emitted at low inclination could be scattered into a high-inclination line of sight by polar material, as in the classical interpretation of Seyfert 2 polarization (Antonucci & Miller 1985; Smith et al. 2004). Unlike in a Seyfert 2 nucleus, the attenuated direct equatorial continuum would remain visible and dilute the scattered component. In our SED models, the low-inclination optical continuum is brighter than the edge-on continuum by a factor of approximately 5–10 (Madau 2026). Redirecting an effective 10% of this radiation toward the observer could therefore make the scattered flux comparable to the direct equatorial flux and, for an angle-averaged scattered-light polarization of approximately 30%, produce an observed polarization of approximately 10–15%, neglecting host-galaxy dilution and vector cancellation. Such a high polarization of the scattered light

is possible for optically thin scattering near right angles, but depends on the opening angle, optical depth, and scattering phase function. An unattenuated scattered component this strong would be too blue to preserve the red optical colors of an LRD and would therefore also have to suffer substantial extinction along its path to the observer, although not necessarily through the same dust screen as the direct continuum. The argument applies to the anisotropic continuum and does not apply to the spatially extended, isotropically-emitted narrow-line radiation.

Two limitations are central. First, the Chandrasekhar law is an idealized closure for a semi-infinite, plane-parallel, conservative atmosphere. The funnel wall is curved, has spatially varying optical depth, and receives anisotropic irradiation. Our scalar Lambertian wall-to-wall coupling followed by an angle-dependent polarized escape calculation therefore yields approximate polarization fractions; a direction-dependent polarized transfer calculation is needed for precision predictions. Second, the present optical baseline is too short to constrain the weak color dependence or

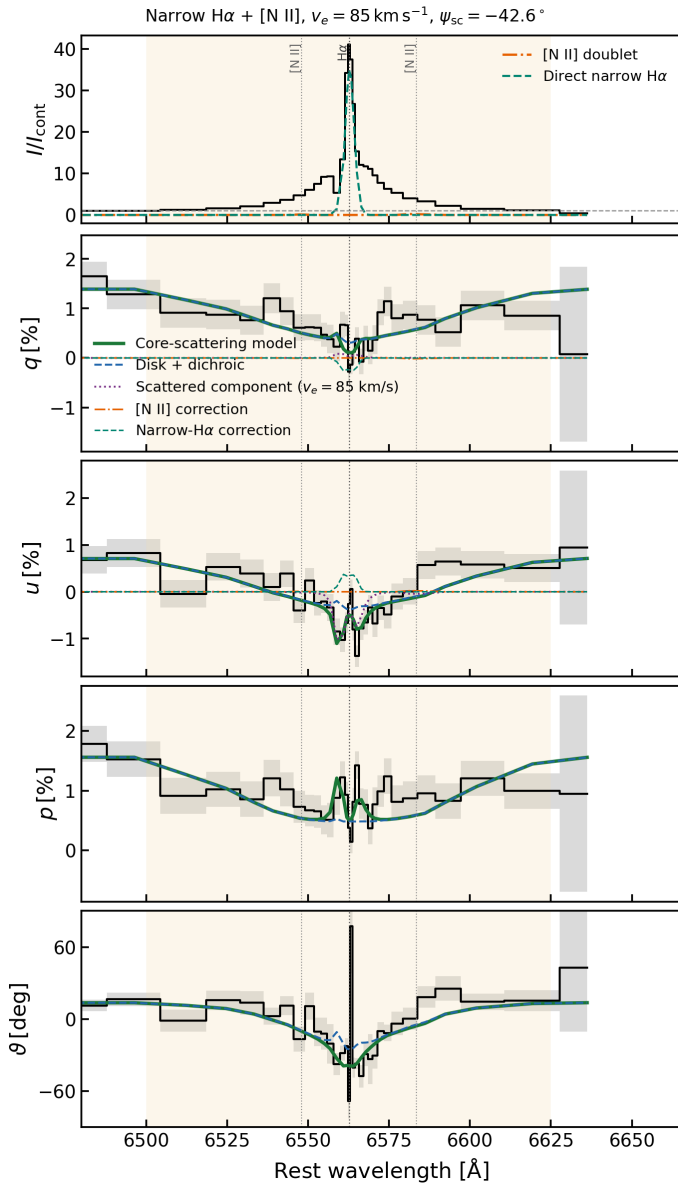


Fig. 6. Polarized narrow-component fit to the $H\alpha + [N II]$ core. The top panel shows the continuum-normalized intensity and direct narrow-line components. Lower panels show q , u , p , and the polarization position angle θ : black histograms and gray bands are the data and 1σ uncertainties, green is the extended model, and dashed blue is the disk-plus-dichroic baseline. Teal and orange curves in the q and u panels show the removal of dichroic polarization over the direct narrow-line flux; separate component amplitudes are not shown in the nonlinear p and θ panels. The position angle is poorly constrained where p is small. Shading marks the fitted window and dotted lines the three rest wavelengths.

separate $\dot{m}$ from i . The observed continuum slopes are statistically consistent with the nearly gray best-fitting model.

A broader wavelength range offers further tests. UV or blue spectropolarimetry can probe the relative roles of nearly gray electron scattering and wavelength-dependent dichroic polarization, while red or near-infrared data can trace the declining dichroic tail and the less-extinguished Paschen lines. The comparison is not unique, however: dichroic polarization can have different wavelength dependences for different grain populations and need not follow

the single Serkowski curve adopted here. In addition, UV emission from young stars could dilute the AGN polarization or contribute a separate Stokes vector if it is itself dichroically polarized. The unification model further predicts that LRDs should, on average, be more polarized than unobscured LBDs, with the largest rest-frame UV polarization in the most inclined and rapidly accreting sources. Conversely, low-inclination objects should remain weakly polarized because of cancellation.

In summary, the continuum polarization of J1025+1402 is consistent with a geometrically thick, rapidly accreting flow seen at high inclination and combined with modest dichroic transmission. The narrow-line residuals provide tentative evidence for a separate dusty scattering region. Spectropolarimetry therefore supplies a geometric test of the orientation-based little-dot model that is complementary to SED fitting and emission-line diagnostics.

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Appendix A: A conical toy model

It is useful to illustrate the geometric origin of the net polarization signal. At each surface element (r, ϕ) , the local polarization direction is perpendicular to the local meridional plane defined by the surface normal $\hat{n}$ and the observer's line of sight $\hat{k}$. As ϕ runs from 0 to 2π at fixed r , this direction rotates on the plane of the sky, so the polarization angle $\chi(r, \phi; i)$ varies and the factor $\cos 2\chi$ changes sign around the ring. At exactly face-on inclination, μ and $p(\mu)$ are independent of ϕ , while the polarization direction is tangential at every azimuth. The azimuthally integrated Stokes vector therefore vanishes exactly. As the inclination increases, the azimuthal dependence of both the projected weights and the polarization directions breaks this cancellation, producing a finite net polarization. The observed polarization fraction is thus the small residual left after vector cancellation over the visible surface, not the local Chandrasekhar polarization of an individual surface element.

This behavior can be illustrated by considering a circular ring on a conical funnel wall of half-opening angle Θ , measured from the symmetry axis. We neglect non-local self-shadowing. For the surface normal directed into the funnel cavity, we write

$$\hat{n} = (-\cos \Theta \cos \phi, -\cos \Theta \sin \phi, \sin \Theta), \quad (\text{A.1})$$

while the observer direction is

$$\hat{k} = (\sin i, 0, \cos i). \quad (\text{A.2})$$

The local emission-angle cosine is therefore

$$\mu(\phi; i, \Theta) = \hat{n} \cdot \hat{k} = \sin \Theta \cos i - \cos \Theta \sin i \cos \phi. \quad (\text{A.3})$$

For $i > \Theta$, μ becomes negative near $\phi = 0$. These elements are back-facing and are excluded by the local visibility condition $\mu > 0$. This is distinct from non-local self-shadowing, in which the ray from an otherwise visible element to the observer intersects another part of the funnel; the latter effect is omitted in this toy model.

At exactly face-on inclination, $\mu = \sin \Theta$ is independent of azimuth, so every surface element has the same local Chandrasekhar polarization $p(\sin \Theta)$. The projected polarization direction nevertheless rotates around the ring. For $i > 0$, choosing the projected symmetry axis as the reference direction on the sky gives

$$\cos 2\chi = 2 \frac{\cos^2 \Theta \sin^2 \phi}{1 - \mu^2} - 1. \quad (\text{A.4})$$

Taking the face-on limit, $i \rightarrow 0$, yields

$$\cos 2\chi = 2 \sin^2 \phi - 1 = -\cos 2\phi, \quad (\text{A.5})$$

and hence

$$\begin{aligned} Q_{\nu, \text{sky}} &\propto p(\sin \Theta) I_{\nu}(r, \sin \Theta) \sin \Theta \int_0^{2\pi} \cos 2\chi \, d\phi = 0, \\ U_{\nu, \text{sky}} &= 0. \end{aligned} \quad (\text{A.6})$$

Thus the local polarization may be several percent even though the surface-integrated polarization vanishes exactly by azimuthal cancellation.

For a small but finite inclination satisfying $i < \Theta$, all azimuths remain locally visible and

$$\begin{aligned} \mu(\phi; i, \Theta) &\simeq \sin \Theta - \cos \Theta i \cos \phi \\ &\quad - \frac{1}{2} \sin \Theta i^2 + \mathcal{O}(i^3), \quad i \ll 1. \end{aligned} \quad (\text{A.7})$$

Both the projected weights and the sky-projected polarization directions then acquire an azimuthal dependence. To isolate this geometric effect, we hold $p(\mu)$ and $\mathcal{H}(\mu)$ fixed at their face-on values. Reflection symmetry gives $U_{\nu, \text{sky}} = 0$. Defining the observed polarization fraction of the conical ring as

$$p_{\text{obs}} \equiv \frac{\sqrt{Q_{\nu, \text{sky}}^2 + U_{\nu, \text{sky}}^2}}{f_{\nu, \text{obs}}}, \quad (\text{A.8})$$

we obtain

$$p_{\text{obs}} = p(\sin \Theta) \left| \frac{\int_0^{2\pi} \mu(\phi; i, \Theta) \cos 2\chi \, d\phi}{\int_0^{2\pi} \mu(\phi; i, \Theta) \, d\phi} \right|. \quad (\text{A.9})$$

Expanding to the first non-vanishing order in inclination gives

$$p_{\text{obs}} \simeq \frac{3}{4} p(\sin \Theta) i^2, \quad i \ll 1, \quad (\text{A.10})$$

with i expressed in radians. The net polarization seen by the observer is therefore quadratically suppressed at low inclination even though every surface element retains a finite local Chandrasekhar polarization. In this simplified limit, the funnel opening angle enters only through the local value $p(\sin \Theta)$. Equation (A.10) explicitly shows that p_{obs} is the residual after vector cancellation over the visible surface. At larger inclinations, this cancellation becomes progressively less effective until local visibility and, in the full disk calculation, self-shadowing become important.