Little red dots as a cosmological probe: constraining H_0 with quasi-periodic pulsations

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(Dated: June 5, 2026)

The James Webb Space Telescope (JWST) has uncovered a population of "little red dots" (LRDs) at $z \gtrsim 4$, potentially representing early supermassive black holes embedded in dense gaseous envelopes. The recent discovery of the lensed LRD R2211-RX1 reveals significant variability on rest-frame timescales of decades, which may be interpreted as quasi-periodic variation that has a potential physical parallel to stellar pulsations. In this work, we derive an idealized, self-consistent period-luminosity-temperature (P-L-$T_{\rm eff}$) relation based on the hydrostatic envelope model. If this theoretical relation holds and can be empirically validated/calibrated, it would offer a novel framework for constraining the Hubble constant (H_0). The current sparse sampling of R2211-RX1 yields a preliminary $H_0 = 120.7^{+47.0}_{-46.5}$ km s^{-1} Mpc^{-1} as a proof-of-concept, with the error budget dominated by the uncertainty of the pulsation period. Our forecasting analysis shows that continuous monitoring over a 10-year baseline can reduce the H_0 uncertainty to 3–20%, depending on the intrinsic pulsation period, while the systematic uncertainty floor remains to be fully characterized. This method offers a potential independent probe to measure luminosity distances in the early universe.

I. INTRODUCTION

James Webb Space Telescope (JWST) has revealed a ubiquitous population of compact [1, 2], high-redshift ($z \gtrsim 4$) sources characterized by unique V-shaped spectral energy distributions (SEDs), widely known as "little red dots" [LRDs; 3–5]. While their compactness and broad Balmer emission lines suggest accreting supermassive black holes [SMBH; 4, 6–9], LRDs also exhibit puzzling characteristics compared with typical active galactic nuclei (AGNs), such as the unusual faintness in X-ray [10–12], radio [13], and observed mid-to-far infrared bands [2, 14, 15], and the little to no short-term variability [16–19]. They may represent a transitional or previously unexplored phase of BH growth in the early universe [20].

Compelling evidence increasingly suggests that LRDs are AGNs cocooned within dense, optically thick gaseous envelopes [21–24]. In this scenario, the central AGN radiation is reprocessed with an envelope with a surface temperature of $T_{\rm eff} \simeq 4000$–6000 K, similar to the typical photospheric temperature of giant stars. This dense gas envelope naturally explains the red optical continua, prominent Balmer breaks, and a high fraction of absorption lines on LRD spectra [8, 24–27]. This "BH-envelope" hypothesis has gained significant support from the discovery of local LRD analogs at $z \simeq 0.1$–0.2, like J1025+1402 [28, 29]. It displays the full set of LRD characteristics, together with strong absorption features from low-ionization metal species such as the Ca II triplet, Na I D, and K I. These spectral fingerprints suggest that the gas properties at the LRD envelope surface are remarkably similar to those of yellow supergiants.

The BH-envelope model naturally predicts quasi-periodic variability driven by κ-mechanism pulsations in radiation-pressure-dominated[30, 31], physically analogous to stellar pulsations giant stars [32–34] and in accreting massive stars in the present-day universe [35, 36], a short-lived stage before collapsing into heavy BH seeds [37, 38]. Zhang *et al.* [39] recently provided evidence for such variability in a gravitationally lensed LRD R2211-RX1, where ~ 130-year time delay between images reveal intrinsic brightness and color variations consistent with blackbody temperature and radius fluctuations over a rest-frame period of decades. The pulsation nature of this variability is further supported by the instability analysis of Cantiello *et al.* [30], who demonstrated that such gas envelopes surrounding an accreting BH [i.e., a quasi-stellar structure; 40] are unstable to radial pulsations driven by the κ-mechanism within a specific instability strip.

Such pulsations may offer a novel window to constrain the Hubble constant (H_0), helping to address the emerging "Hubble tension" between early- and late-universe measurements [41–43]. This tension signals potential unresolved systematic effects or new physics beyond the standard model [43], highlighting the urgent need for independent cosmological probes. If pulsating LRDs follow

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a robust period-luminosity (P-L) relation analogous to the Leavitt Law for Cepheids [44], they could serve as independent "standard candles" to measure luminosity distance at high redshifts. In this Letter, we establish an idealized, self-consistent $P - L - T_{\rm eff}$ framework for LRD pulsations and derive its direct relation to H_0. We then apply this model to R2211-RX1 and demonstrate through forecasting that a modest sample of LRDs can constrain H_0 to within $\lesssim 10\%$, providing a unique cosmological probe at high redshifts.

II. PHYSICAL MODEL AND COSMOLOGICAL INFERENCE FRAMEWORK

A. LRD Pulsation Model

The purpose of this subsection is not to present a competitive measurement of the Hubble constant. We instead aim to establish a self-consistent theoretical framework linking pulsation observables of LRDs to cosmological distances and to identify the dominant sources of uncertainty relevant for future observations.

Some LRDs exhibit deterministic variability consistent with radial pulsation, which can be described by

$$P_{\rm src} = 2\pi Q \left(\frac{R^3}{GM}\right)^{1/2}, \tag{1}$$

where M and R denote the BH mass and the photospheric radius. Q is the pulsation constant characterized by the mechanical and thermal structure of the envelope (see below). Since the photospheric emission is powered by mass accretion onto the central BH, we parameterize the time-averaged luminosity of the radiation-pressure-supported envelope in quasi-hydrostatic equilibrium as

$$L = (1-\beta)L_{\rm Edd} = (1-\beta)\frac{4\pi cGM}{\kappa_{\rm es}}, \tag{2}$$

where $\kappa_{\rm es}$ is the Thomson scattering opacity and $\beta = P_{\rm gas}/(P_{\rm gas}+P_{\rm rad})$. For an envelope in hydrostatic equilibrium with a polytropic index $n = 3$, one obtains $1-\beta \approx 0.003\ (\mu\beta)^4(M/M_\odot)^2$ [45, 46], leading to

$$\beta \approx 0.00256 \left(\frac{\mu}{0.6}\right)\left(\frac{M}{10^6\ M_\odot}\right)^{-1/2}. \tag{3}$$

Therefore, β is not an independent, free parameter but is determined by the BH mass. For such a radiation-pressure-dominated envelope, where the specific heat index approaches $\gamma_{\rm ad} \simeq 4/3 + \beta/6$ and a constant entropy profile is maintained due to efficient convection, the pulsation constant is computed as $Q = 0.3330$ for the first overtone mode [47] [48]. Cantiello *et al.* [30] found that the pulsation constant is $Q = 0.3226$ for the first overtone mode in the quasi-stellar configuration. The two values are consistent at a $\sim 3\%$ error. We adopt the latter value in the following analysis ($Q_0 = 0.3226$).

Assuming that the emission follows a blackbody spectrum with an effective temperature $T_{\rm eff}$, the luminosity is given by

$$L = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4, \tag{4}$$

where $\sigma_{\rm SB}$ is the Stefan–Boltzmann constant. While the envelope pulsates, Eqs. (1) and (2) characterize its time-averaged state, with Eq. (4) remaining valid for both instantaneous and average values. Combining Eqs. (1)–(4), we obtain a period–luminosity–temperature relation:

$$\begin{aligned} P_{\rm src} &= Q_0\left(\frac{4\pi^3 c^2}{\kappa^2\sigma_{\rm SB}^3}\right)^{1/4}(1-\beta)^{1/2}L^{1/4}T_{\rm eff}^{-3}, \\ &= 9.74\ {\rm yr}\ (1-\beta)^{1/2}\left(\frac{L}{10^{10}\ L_\odot}\right)^{1/4}\left(\frac{T_{\rm eff}}{5000\ {\rm K}}\right)^{-3}. \end{aligned} \tag{5}$$

This decade pulsation period is accessible for relatively long-term monitoring, especially for low-redshift LRDs (e.g., $z < 1$).

Eq. (5) provides an idealized theoretical baseline, though complexities—akin to classical pulsating variables—may arise. Factors such as the pulsation constant Q [33] and potential dependence on the envelope metallicity could introduce systematic deviations [49, 50]. While future empirical calibration is essential to refine this relation into a precision cosmological tool, this model provides a robust approximation. In the following sections, we adopt this framework to demonstrate the potential of LRDs as an independent H_0 probe.

B. Constraint on H_0

1. Direct Period Measurements

The luminosity distance is related to the observed flux as $D_L = (L/4\pi F_{\rm obs})^{1/2}$, and thus can be expressed as

$$D_L = \frac{\kappa_{\rm es}F_{\rm obs}}{c(1-\beta)}\left(\frac{P_{\rm src}}{2\pi Q_0}\right)^2\left(\frac{\sigma_{\rm SB}T_{\rm eff}^4}{F_{\rm obs}}\right)^{3/2}. \tag{6}$$

Using the definition of the luminosity distance and $P_{\rm obs} = (1+z)P_{\rm src}$, we obtain

$$H_0 = (1+z)^3\frac{c^2(1-\beta)}{\kappa_{\rm es}}\cdot\frac{\langle F_{\rm obs}\rangle^{1/2}}{\langle\sigma_{\rm SB}T_{\rm eff}^4\rangle^{3/2}}\left(\frac{P_{\rm obs}}{2\pi Q_0}\right)^{-2}\int_0^z\frac{dz'}{E(z')}, \tag{7}$$

where all observed quantities are defined as time-averaged values over one pulsation period,

$$\langle A\rangle = \frac{1}{P_{\rm obs}}\int_0^{P_{\rm obs}} A(t)dt. \tag{8}$$

This treatment is required because the relations above are defined with global equilibrium quantities rather than instantaneous values.

The uncertainty of the Hubble constant measurement using Eq. (7) is quantified as

$$\frac{\delta H_0}{H_0} \simeq \sqrt{\frac{1}{4}\left(\frac{\delta\langle F_{\rm obs}\rangle}{\langle F_{\rm obs}\rangle}\right)^2 + \frac{9}{4}\left(\frac{\delta\langle T_{\rm eff}^4\rangle}{\langle T_{\rm eff}^4\rangle}\right)^2 + 4\left(\frac{\delta P_{\rm obs}}{P_{\rm obs}}\right)^2}. \tag{9}$$

Since $\beta \ll 1$ is fixed by the hydrostatic-equilibrium structure of the envelope, its fractional uncertainty of $(1-\beta)$ is expected to be subdominant compared to observational errors and is neglected here. The Q_0 value may have uncertainty of a few percent, but its contribution to the H_0 uncertainty is currently sub-dominant as discussed latter.

An alternative H_0 formulation via surface gravity g avoids assuming an Eddington ratio (Appendix B), but we prioritize the current formulation for our subsequent analysis due to lack of constraints on $\log g$.

2. Reconstruction of Pulsation Periods

Long-term time-domain monitoring in a decade is feasible for low-redshift LRDs. For high-redshift sources, on the other hand, the observed period is further stretched by cosmic time dilation, $P_{\rm obs} = (1+z)P_{\rm src}$, making direct period measurements extremely expensive in terms of observational baseline.

An important exception arises when high-redshift LRDs are gravitationally lensed and produce multiple images. In such systems, the intrinsic pulsation signal is replicated in each image, but shifted in time due to gravitational lensing time delays and rescaled in flux by different lensing magnifications. By correcting both the time delays and the flux magnification, the intrinsic light curves can be reconstructed without requiring continuous monitoring over the full observed-frame period [e.g., the R2211-RX1 and R2211-RX2 in RXC J2211–0350, where the observed-frame baseline reaches ~ 130 years; 39].

In the thin-lens approximation, the arrival time of a light ray observed at angular position $\boldsymbol{\theta}$ is given by the lensing potential $\psi(\boldsymbol{\theta})$ as

$$t(\boldsymbol{\theta}) = \frac{1+z_l}{c}\frac{D_{\rm ol}D_{\rm os}}{D_{\rm ls}}\left[\frac{1}{2}|\boldsymbol{\theta}-\boldsymbol{\beta}_{\rm s}|^2 - \psi(\boldsymbol{\theta})\right], \tag{10}$$

where $\boldsymbol{\beta}_{\rm s}$ is the unlensed source position and z_l is the redshift of the lens. $D_{\rm ol}$, $D_{\rm os}$, and $D_{\rm ls}$ are the angular diameter distances between the observer, lens, and source, respectively. The expression in the bracket depends only on the lens mass model and is independent of cosmology. Defining the time-delay distance as

$$D_{\Delta t} \equiv (1+z_l)\frac{D_{\rm ol}D_{\rm os}}{D_{\rm ls}} \propto H_0^{-1}, \tag{11}$$

the reconstructed pulsation period in the observer's frame scales as $P_{\rm obs} \propto D_{\Delta t} \propto H_0^{-1}$. Thus, in lensed systems, the pulsation period itself becomes a cosmological observable through the time-delay distance.

Introducing the Fermat potential, $\tau \equiv |\boldsymbol{\theta}-\boldsymbol{\beta}_{\rm s}|^2/2 - \psi(\boldsymbol{\theta})$, the fractional uncertainty in the Hubble constant can be expressed as

$$\frac{\delta H_0}{H_0} \simeq \sqrt{\frac{1}{4}\left(\frac{\delta\langle F_{\rm obs}\rangle}{\langle F_{\rm obs}\rangle}\right)^2 + \frac{9}{4}\left(\frac{\delta\langle T_{\rm eff}^4\rangle}{\langle T_{\rm eff}^4\rangle}\right)^2 + 4\left(\frac{\delta\tau}{\tau}\right)^2}. \tag{12}$$

Note that $\delta\langle F_{\rm obs}\rangle/\langle F_{\rm obs}\rangle$ here, and in subsequent lensing cases, accounts for the magnification uncertainties derived from the lens mass model. This expression clarifies that, for lensed cases, the dominant additional uncertainty arises from the reconstruction of the Fermat potential, i.e., the lens mass model, rather than from the intrinsic pulsation period itself. Consequently, gravitational lensing provides a unique pathway to constrain the period even for sources at cosmological distances.

Multi-epoch monitoring of lensed images can extend the temporal baseline but introduces a mixture of cosmology-dependent time delays and cosmology-independent time intervals from the monitoring, therefore breaking the simple analytical relation. To resolve this complexity, we employ a maximum likelihood analysis to constrain H_0, with details provided in Appendix A.

III. OBSERVATIONAL CONSTRAINS AND FUTURE PROSPECTS

A. Constraints from the Lensed LRD R2211-RX1

Strong gravitational lensing of LRDs by massive galaxy clusters provides a unique opportunity to study the variability of high-redshift LRDs over decades, leveraging time delays between multiple images. To date, four multiply imaged LRDs have been reported in the literature [1, 39, 51, 52]. Among these, R2211-RX1, recently identified by Zhang *et al.* [39], is uniquely the only known LRD exhibiting clear pulsation signatures. With a predicted time delay exceeding a century, R2211-RX1 allows for full-cycle light curve sampling to constrain the pulsation period. We use this object to demonstrate our cosmological inference framework.

We adopt a Monte Carlo (MC) approach to propagate the uncertainties from the observable parameters, $P_{\rm src}$, $T_{\rm eff}$, and $F_{\rm obs}$, into H_0. The source pulsation period is determined as $P_{\rm src} = 40.8^{+6.7}_{-8.9}$ yr through multi-band sinusoidal light-curve modeling following Zhang *et al.* [39]. The effective temperature and bolometric flux are derived by fitting the continuum SED with a blackbody plus power-law model, yielding $\langle T_{\rm eff}\rangle = 3834^{+77}_{-80}$ K and $\langle F_{\rm obs}\rangle = 8.87^{+0.25}_{-0.22} \times 10^{-16}$ erg s^{-1} cm^2. Currently, the $\sim 25\%$ uncertainty in $P_{\rm src}$ dominates the total error budget, primarily due to the sparse sampling of the light curves. Detailed descriptions of the Bayesian inference and prior constraints are provided in Appendix C.

To isolate the H_0 dependence in $P_{\rm obs}$, we define $P_{\rm obs} = \mathcal{P}_z H_0^{-1}$, where $\mathcal{P}_z$ a quantity primarily determined by the lens model. Then we obtain

$$H_0 = \frac{\kappa_{\rm es}}{c^2(1-\beta)(1+z)^3} \frac{\langle \sigma_{\rm SB} T_{\rm eff}^4 \rangle^{3/2}}{\langle F_{\rm obs} \rangle^{1/2}} \times \left(\frac{\mathcal{P}_z}{2\pi Q_0} \right)^2 \left(\int_0^z \frac{dz'}{E(z')} \right)^{-1} . \tag{13}$$

While $\mathcal{P}_z$ retains a minor redshift dependence through the ratio $D_{\rm os}/D_{\rm ls}$, this effect is negligible when the source redshift significantly exceeds that of the lens. We derive the posterior distribution of $\mathcal{P}_z$ by multiplying $P_{\rm obs}$ by the fiducial H_0 used in the lens model.

Finally, we obtain an estimation of $H_0 = 120.7^{+47.0}_{-46.5}$ $\rm km\,s^{-1}\,Mpc^{-1}$. Although the central value is high, it remains consistent with local measurements within the 1σ uncertainty ($\sim 40\%$), which is primarily driven by the rest-frame period constraint. Despite the current limited precision, this result serves as a critical proof of concept for using LRD pulsations as an independent cosmological probe.

B. Physical Validation and Systematic Uncertainties

Future JWST multi-cycle spectrophotometric monitoring will be essential to validate the pulsational nature of LRDs and the P-L-$T_{\rm eff}$ relation, establishing LRD pulsations as a robust cosmological probe. Beyond periodic light curves, the pulsation predicts periodic envelope expansion and contraction, which should manifesting as ~ 100 km s^{-1} velocity shifts in absorption lines (e.g., Hα, He Iλ10830). Such spectroscopic signatures, analogous to the atmospheric dynamics of pulsating stars, would provide kinematic evidence independent of photometry [39]. Furthermore, expanding the sample with additional lensed or low-redshift LRDs will allow empirical calibration of the relation, anchoring LRDs as a reliable H_0 probe.

Assuming the physical scaling holds, the precision of this cosmological framework is governed by the pulsation period $P_{\rm src}$ determination and the potential systematic departures from the idealized P-L-$T_{\rm eff}$ relation. The current theoretical framework assumes specific values for the Q_0 and β. Under the self-consistent assumption that the gas envelope is supported by radiation pressure, $\beta \ll 1$ is naturally constrained, minimizing its error contribution. Assigning a conservative 5% theoretical uncertainty (reflecting variations in envelope geometry or polytropic index) to the pulsation constant Q_0 yields an H_0 systematic floor of $\sim 10\%$. Currently, the $\sim 25\%$ uncertainty in $P_{\rm src}$ dominates the error budget. The systematic floor will only become limiting after $\gtrsim 5$ years of monitoring constrains the period to high precision (Fig. 1). Future detailed modeling and empirical calibration using a larger sample of lensed and/or low-redshift LRDs will be essential to precisely characterize these systematic terms. In particular, low-redshift LRDs offer a unique laboratory, providing both high signal-to-noise data and the observational feasibility for the high-cadence, long-term monitoring required to empirically calibrate parameters such as Q_0. A well-calibrated local framework will significantly reduce the systematic floor when applying this method to the high-redshift population discovered by JWST.

Additional uncertainties stem from $T_{\rm eff}$ and $F_{\rm obs}$, which will only become a limitation once $P_{\rm src}$ is precisely constrained and the systematic uncertainties of the P-L-$T_{\rm eff}$ are well understood. Current analysis for R2211-RX1 relies only on JWST/NIRCam photometry, where the optical SED can suffers from a degeneracy between $T_{\rm eff}$, dust extinction, and the Balmer break strength, potentially introducing systematic biases [55]. Future multiwavelength observations will help break these degeneracies: JWST/NIRSpec prism spectroscopy can better constrain the continuum and Balmer break strength, while JWST/MIRI photometry can isolate dust extinction through the long-wavelength SED slope.

Finally, for lensed sources, the inferred H_0 is subject to the mass-sheet degeneracy [56]. An external convergence $\kappa_{\rm ext}$ scales the observed flux as $F_{\rm obs} \propto (1-\kappa_{\rm ext})^2$ and the reconstructed pulsation period as $P_{\rm obs} \propto (1-\kappa_{\rm ext})^{-1}$ [57]. Consequently, the H_0 derived from Eq. (13) would be affected as $H_0 \propto (1-\kappa_{\rm ext})$. This systematic effect can be effectively quantified and mitigated by characterizing the lens environment and line-of-sight mass distribution using high-resolution imaging [58].

C. Forecast for H_0 Constraints

We now assess the achievable constraints on H_0 from this framework once it is validated, assuming continuous monitoring of R2211-RX1 over the next 10 years and a true $P_{\rm src} = 31.8$ years. We ignore the complexity due to the mixture of lensing time delays and direct monitoring here. We consider three epochs: the completion of Cycles 5/6 JWST program GO-10005 (+1, +1.5, +2, and +2.5 yr baseline), and extended monitoring at 5 and 10 years (e.g., Cycles 10 and 15). For each configuration, we perform 50 independent realizations, incorporating random Gaussian noise into the mock datasets to match current measurement uncertainties.

As shown in Fig. 1 (left), the posterior distribution for the rest-frame period $P_{\rm src}$ sharpens significantly as the monitoring baseline extends. The uncertainty reduces to < 5 yr after 5 years of monitoring, even reaching sub-year precision for shorter intrinsic periods (see Appendix D). Assuming a 3% uncertainty in the structure constant Q_0 and mitigated systematics ($\sim 1\%$ statistical noise) in $F_{\rm obs}$ and $T_{\rm eff}$ through extensive observations, we project the cosmological impact in Fig. 1 (right). The upper panel illustrates the expected convergence of the H_0 PDF toward the fiducial value. In the lower panel, we compare the precision against standard benchmarks from SH0ES [42], Planck [41], tip of the red giant branch [TRGB method; 53], and strong gravitational lensing [54].

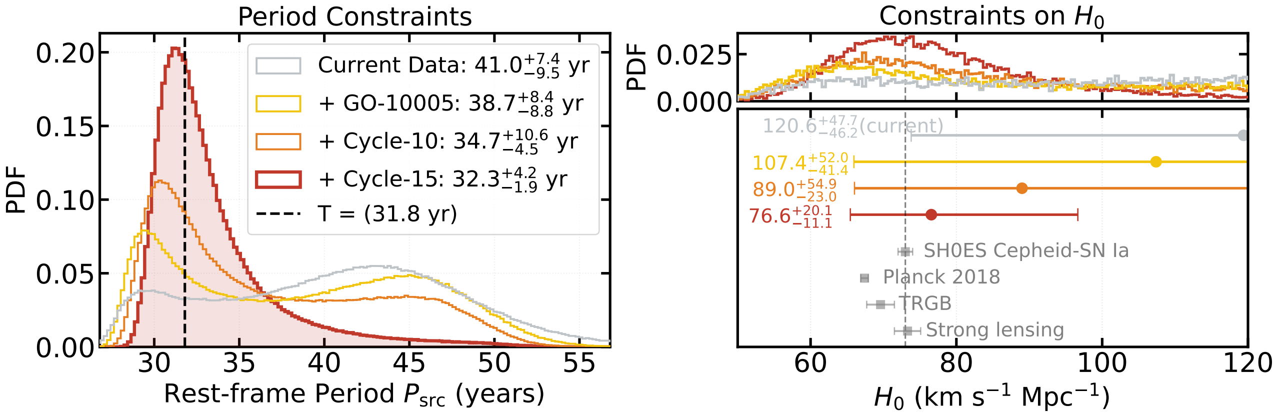

FIG. 1. Left: Posterior probability density functions (PDF) for the rest-frame period $P_{\rm src}$. We consider the approved program GO-10005 and two extended scenarios incorporating additional monitoring over 5-year and 10-year baselines. The uncertainty σ_P diminishes with extended monitoring, converging toward the assumed value of 31.8 years. Right: The corresponding constraints on the Hubble constant H_0. Top panel shows the aggregated PDF of H_0 derived from different period measurements (correspond to the color in the left panel), showing the significant decrease in uncertainties as monitoring accumulated. Bottom panel shows comparison of H_0 measurement precision across different scenarios. The colored points represent our simulated results with 1σ uncertainties. Existing constraints (gray squares) including SH0ES [42], Planck [41], TRGB [53], and Strong Lensing [54] are shown for comparison. The vertical dashed line $H_0 = 73$ km s^{-1}Mpc^{-1}.

Our forecast indicates that with an extended monitoring campaign, a single multiply imaged LRD can achieve a competitive measurement of H_0 with an uncertainty of $\sim$ 15%, potentially reaching a uncertainty level of $\lesssim 5\%$ in the most optimistic case (intrinsically short period). This result indicates that long-term time-domain monitoring is the key observational requirement for this method.

Additionally, in the future, if a sample of $N^{\rm var}_{\rm LRD}$ LRDs exhibiting measurable pulsations becomes available, the uncertainty is expected to decrease as

$$\begin{aligned}\frac{\delta H_0}{H_0} &\simeq \frac{2}{\sqrt{N^{\rm var}_{\rm LRD}}}\frac{\delta P_{\rm obs}}{P_{\rm obs}} \\ &\sim 6.3\% \left(\frac{N^{\rm var}_{\rm LRD}}{10}\right)^{-1/2}\left(\frac{\delta P_{\rm obs}/P_{\rm obs}}{0.1}\right).\end{aligned} \tag{14}$$

Therefore, even a modest sample of $N^{\rm var}_{\rm LRD} \gtrsim 4$ well-characterized pulsating LRDs would allow constraints at the $\lesssim 10\%$ level.

IV. SUMMARY

In this work, we propose a novel cosmological framework linking the quasi-periodic variability of LRDs to distance measurements via a physically motivated P–L–$T_{\rm eff}$ relation for idealized radiation-pressure-supported envelopes. Applying this framework to the lensed LRD R2211-RX1, we obtain a preliminary constraint of $H_0 = 120.7^{+47.0}_{-46.5}$ km s^{-1} Mpc^{-1}. Although the uncertainty remains large, it is dominated by the limited constraint on the pulsation period and serves as a critical proof-of-concept. Our forecast indicates that 10 years of monitoring can reduce H_0 statistical uncertainty to 3–20%, depending on the intrinsic period.

The ultimate accuracy of this framework depends on quantifying systematic uncertainties in the P–L–$T_{\rm eff}$ relation, which remain to be fully characterized. Empirical calibration via low-redshift LRDs and improved modeling are essential to mitigate these effects and validate the theoretical baseline. Once calibrated, this method will provide a robust, independent H_0 probe. Though nascent, this approach offers a complementary test for cosmological models, particularly at high redshifts where standard candles are scarce.

ACKNOWLEDGMENTS

We acknowledge support from the National Natural Science Foundation of China (12225301, 12573015, W2532003), the Beijing Natural Science Foundation (IS25003), and the China Manned Space Program (CMS-CSST-2025-A09). M.O. acknowledges support from JSPS KAKENHI Grant Numbers JP25H00662 and JP22K21349. We thank the JWST VENUS team, led by Seiji Fujimoto and Dan Coe, for the execution of the program (GO-6882) and for their commitment to making the data publicly available. We thank Subo Dong and Cheng Zhao for helpful discussions.

Appendix A: Likelihood Analysis Framework for Multi-epoch Observations of Lensed Systems

To self-consistently constrain H_0 for cases with a mixture of H_0-dependent (lensing-derived) and H_0-

independent (monitoring-derived) timescales, we can employ a maximum likelihood analysis. This approach treats the observed data from all multiple images as a single realization of the source's intrinsic light curve, transformed by the lens. We define a likelihood function $\mathcal{L}(H_0, \vec{\theta})$ that incorporates both the lens model parameters and the pulsation model:

$$\ln \mathcal{L}(H_0, \vec{\theta}) = -\frac{1}{2} \sum_{j=1}^{N_{\rm img}} \sum_{i=1}^{N_{\rm obs,j}} \left[\frac{\left(F_{i,j}^{\rm obs} - \mu_j F_{\rm model}(t_i - \Delta t_j(H_0); \vec{\theta}_{\rm var}) \right)^2}{\sigma_{i,j}^2} \right] + \ln \mathcal{P}(\vec{\theta}) \tag{A1}$$

Each term in this expression is defined as follows:

- $F_{i,j}^{\rm obs}$ and $\sigma_{i,j}$: The observed flux and its associated uncertainty for the i-th observation of the j-th lensed image at observed time t_i.
- μ_j: The magnification factor for image j. These are treated as nuisance parameters constrained by the lens mass model.
- $\Delta t_j(H_0)$: The relative time delay for image j with respect to the reference image. Crucially, the cosmology dependence can be explicitly isolated in the time delay $\Delta t_j(H_0) = \frac{D_{\Delta t}(H_0)}{c} \Delta\tau_j$, where $\Delta\tau_j$ is the Fermat potential difference relative to the leading image. $\Delta\tau_j$ are treated as nuisance parameters
- $F_{\rm model}$: The template for the intrinsic variability, such as the simple pulsation model $F_{\rm model}(t) = f_0[1 + a\sin(2\pi t/P_{\rm src} + \phi)]$.
- $\vec{\theta}$: The set of all parameters (except H_0) to be explored, which can be subdivided into $\vec{\theta} = \{\vec{\theta}_{\rm var}, \vec{\theta}_{\rm lens}\}$:
  - $\vec{\theta}_{\rm var} = \{P_{\rm src}, f_0, a, \phi, \text{etc}\}$: Parameters describing the intrinsic pulsation, where $P_{\rm src}$ is the rest-frame period.
  - $\vec{\theta}_{\rm lens} = \{\Delta\tau_j, \mu_j\}$: Parameters from the lens mass model, which serve as nuisance parameters and carry their own systematic uncertainties.
- $\ln \mathcal{P}(\vec{\theta})$: The prior distribution, which incorporates independent constraints from the lens model and theoretical limits on LRD pulsation parameters (e.g., Q_0).

The full parameter vector $\vec{\theta}$ encompasses not only the intrinsic LRD variability properties ($\vec{\theta}_{\rm var}$) but also the lens model parameters (μ_j and $\Delta\tau_j$). The second term, $\ln \mathcal{P}(\vec{\theta})$, serves as an informative prior that links the light curve modeling to the lens model. Specifically, the joint posterior distributions of the magnifications and Fermat potentials derived independently from the lens mass model are adopted as the priors $\mathcal{P}(\mu_j, \Delta\tau_j)$. By exploring this full high-dimensional parameter space using MCMC methods, we effectively marginalize over the nuisance parameters inherited from the lens mass model. This approach ensures that the systemic uncertainties of both the magnification and the time delays are robustly propagated into the final posterior constraint on H_0.

Appendix B: Alternative H_0 Formulation via Surface Gravity

From Eqs. (1)–(4), one can also obtain:

$$(GM)^{1/3} = \frac{\kappa_{\rm es}\sigma_{\rm SB}T_{\rm eff}^4}{c(1-\beta)} \left(\frac{P_{\rm src}}{Q}\right)^{4/3} = g\left(\frac{P_{\rm src}}{Q}\right)^{4/3}, \tag{B1}$$

where $g(= GM/R^2)$ is the surface gravity of the photosphere. This gives the second formulation for the Hubble constant:

$$H_0 = \frac{cQ^2(1+z)^3}{gP_{\rm obs}^2} \cdot \frac{\langle F_{\rm obs}\rangle^{1/2}}{\langle \sigma_{\rm SB}T_{\rm eff}^4\rangle^{1/2}} \int_0^z \frac{dz'}{E(z')}. \tag{B2}$$

This expression avoids assuming an Eddington ratio or direct electron scattering opacity by instead linking the physical scale of the photosphere to surface gravity g. While empirical calibrations using features like the CaT equivalent width [EW; 59] can estimate $\log g$, significant uncertainties remain regarding metallicity degeneracy and the applicability of stellar models to LRDs. This limitation may be mitigated with improved LRD atmosphere modeling and more robust spectroscopic diagnostics [55].

Appendix C: Multi-band Light-curve Modeling and SED Fitting

Following Zhang *et al.* [39], we model the multi-band rest-frame light curves with a sinusoidal pulsation: $F_\lambda(t) = f_{0,\lambda}[1 + a_\lambda \sin(2\pi t/P_{\rm src} + \phi)]$, assuming $z = 4.3$ and the fiducial lens model [39]. The redshift will be precisely determined by future spectroscopic observations. The period $P_{\rm src}$ and phase ϕ are shared across all bands, while $f_{0,\lambda}$ and relative amplitudes a_λ are allowed to vary

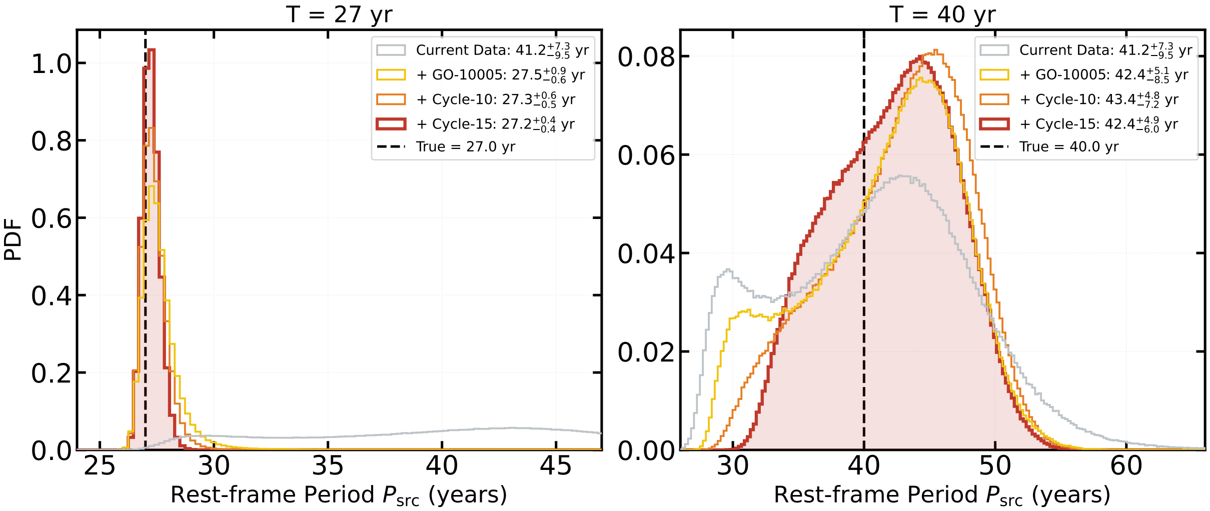

FIG. 2. Posterior probability density functions for the rest-frame period $P_{\rm src}$ with intrinsic $P_{\rm src}$ of 27 years (left) and 40 years (right). The additional monitoring configuration is the same as that in Fig. 1.

by filter. We estimate posteriors using `emcee` [60], with uniform priors $f_{0,\lambda} > 0$ and $a_\lambda < 50\%$. This amplitude limit aligns with pulsational models where non-linear oscillations saturate before reaching extreme amplitudes [30, 61]. Practically, this constraint prevents unphysical, long-period degeneracies arising from sparse sampling, focusing the fit on theoretically predicted decadal modes. With a broad prior on $P_{\rm src} \in [2, 200]$ yr, we obtain $P_{\rm src} = 40.8^{+6.7}_{-8.9}$ yr. The uncertainty in $P_{\rm src}$ accounts for both the reconstruction of the Fermat potential (i.e., the uncertainties of the time delay and magnification from the lens mass model) and the fitting error arising from insufficient sampling. At present, the latter dominates the total uncertainty budget.

For $T_{\rm eff}$ and $F_{\rm obs}$, we model the continuum emission of R2211-RX1 at each epoch as a superposition of a blackbody component and a fixed underlying power-law component, as in [39]. We obtain the covariance between $T_{\rm eff}$ and $F_{\rm obs}$ at each epoch using emcee. Averaging posterior samples across four epochs yields $\langle T_{\rm eff}\rangle = 3834^{+77}_{-80}$ K and $\langle F_{\rm obs}\rangle = 8.87^{+0.25}_{-0.22} \times 10^{-16}$ erg s^{-1} cm^2. These $\sim 2\%$ uncertainties are significantly smaller than that of $P_{\rm src}$, though they remain partially model-dependent.

Appendix D: Period constrains for different intrinsic periods

We also perform the forecast simulation assuming true periods of 27 and 40 years. Fig. 2 shows the forecast PDFs. The constraint on the period is strongly dependent on the intrinsic timescale, with shorter periods yielding tighter constraints and, in some cases, reaching sub-year precision.

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