







## FEADME: Fast Elliptical Accretion Disk Modeling Engine

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### ABSTRACT

We present FEADME (Fast Elliptical Accretion Disk Modeling Engine), a GPU-accelerated Python framework for modeling broad Balmer-line emission using a relativistic elliptical accretion-disk formalism. Leveraging JAX and NumPyro for differentiable forward modeling and efficient Bayesian inference, FEADME enables large-sample, reproducible analyses of disk-dominated emission-line profiles. We apply the framework to 237 double-peaked emitters (DPEs) from the literature and to five tidal disruption events (TDEs) with disk-like H $\alpha$  emission, fitting three physically motivated model families per spectrum and selecting the preferred model using approximate leave-one-out (LOO) cross-validation. We find that AGN exhibit a broad, continuous distribution of disk geometries and kinematics, with significant diversity in disk parameters. Most TDE disk parameter distributions are statistically indistinguishable from those of the AGN, with the sole robust difference being that TDE disks are significantly more circular, **consistent with rapid debris circularization in tidal disruption events**. The majority of both AGN and TDEs favor models that include both a disk and an additional broad-line component, **suggesting that disk emission commonly coexists with more isotropic or wind-driven gas**. These results indicate that once a line-emitting disk forms, its spectroscopic appearance is governed by similar physical processes in both persistent AGN and transient TDE accretion flows, and they demonstrate the utility of FEADME for population-level studies of disk structure in galactic nuclei.

### 1. INTRODUCTION

The innermost regions of active galactic nuclei (AGN) and tidal disruption events (TDEs) host complex, multi-phase gaseous environments shaped by gravity, radiation pressure, magnetic fields, and relativistic motion (N. I. Shakura & R. A. Sunyaev 1973; M. J. Rees 1988; V. I. Pariev et al. 2003; M. Giustini & D. Proga 2019; M. Zhang et al. 2025). The broad Balmer emission lines that arise in these regions are powerful diagnostics of the geometry and kinematics of gas within a few hundred to a few thousand gravitational radii of the central black hole. However, inferring the structure of these regions is challenging given that the broad-line region (BLR) is typically not spatially resolved in all but a few of the nearest and brightest AGN that have recently been probed with optical/infrared interferometry (GRAVITY Collaboration et al. 2024). The BLR encompasses gas spanning a wide range of densities and ionization parameters, with emission arising from rotating disks, disk winds, turbulent or inflowing material, and potentially eccentric or precessing streams. As a result, concrete constraints on BLR structure require models that explicitly incorporate the dynamical and relativistic effects expected in accretion flows.

Double-peaked broad Balmer profiles provide the clearest spectroscopic window into disk-dominated BLR

structure. The classical interpretation, originating with the relativistic Keplerian disk models of K. Chen et al. (1989) and further developed by M. Eracleous et al. (1995); M. Eracleous & J. P. Halpern (2003) and M. Eracleous (1998), demonstrates that geometrically thin, optically thick disks naturally reproduce the characteristic red and blue Doppler peaks and strong asymmetries produced by relativistic boosting. These models predict that disk emission typically originates at radii of tens to hundreds of gravitational radii, well outside the innermost stable circular orbit. Larger samples of double-peaked emitters (DPEs) confirm these expectations and show that disk geometry, kinematics, and emissivity gradients govern the diversity of observed profile morphologies (M. Eracleous et al. 1995; I. V. Strateva et al. 2003; L. C. Ho et al. 2000).

The recent systematic analysis performed by C. Ward et al. (2024) reinforces these findings on population scales. They identify 250 DPEs among ZTF-selected variable AGN, model their Balmer-line profiles using the circular disk model of K. Chen et al. (1989), and demonstrate that the resulting fitted parameters are consistent with disk emission being a ubiquitous component of broad-line AGN. Their results highlight several key points: that the distribution of disk inclinations is consistent with geometric selection effects, that the ob-

served range of disk radii spans orders of magnitudes, and that long-term profile variability in a substantial fraction of sources is well explained by disk asymmetries such as spiral arms or orbiting hotspots (T. Storchi-Bergmann et al. 2003b; J. S. Schimoia et al. 2017).

Disk emission, however, rarely arises from a simple, pristine Keplerian flow. A larger body of theoretical and observational work has shown that radiative transfer through disk atmospheres and outflows can significantly reshape broad-line profiles. Disk-wind models (N. Murray & J. Chiang 1996; D. Proga et al. 2000; L. S. Chajet & P. B. Hall 2013) predict strong anisotropies and the suppression or merging of the two Doppler peaks. H. M. L. G. Flohic et al. (2012) applied a detailed disk-wind radiative transfer framework to Balmer-line profiles and demonstrated that even modest winds can alter line shapes enough to obscure disk signatures that would otherwise be interpreted as classical DPEs. Similarly, E. Bon et al. (2009) investigated the interplay between disk-like emission and the broader BLR using multi-component Gaussian, Lorentzian, and disk models, finding that many AGN with apparently single-peaked profiles may still be consistent with underlying disk emission partially masked by outflows or lower-density gas.

Alternative BLR geometries such as two-phase BLR structures (L. Č. Popović et al. 2004), elliptical disks (M. Eracleous et al. 1995; I. V. Strateva et al. 2003), and precessing eccentric disks (M. Eracleous et al. 1995; T. Storchi-Bergmann et al. 1997) have been invoked to explain asymmetric or transient double-peaked profiles. Elliptical disk models, in particular, enable eccentricity, apsidal orientation, and precession to be incorporated into the velocity field, resulting in line profile shapes that cannot be generated by circular disks alone. They have been widely used both in model AGN and to interpret the physical evolution of long-lived profile asymmetries such as those in Arp 102B or 3C 390.3 (T. Storchi-Bergmann et al. 2003a; S. Gezari et al. 2007; K. T. Lewis et al. 2010; J. S. Schimoia et al. 2012, 2017).

In the context of TDEs, disk-like Balmer emission is seen in a subset of events where debris circularization proceeds efficiently, but many X-ray-bright TDEs with presumably prompt disk formation nonetheless lack obvious disk-like optical line profiles, indicating that the emergence of disk-shaped Balmer emission is not a universal outcome of disk formation. Several recent TDEs have displayed double-peaked or disk-asymmetric  $H\alpha$  profiles (e.g. T. W. S. Holoien et al. 2019; T. Wevers et al. 2022; T. Hung et al. 2020; P. Short et al. 2020; F. K. Liu et al. 2017; N. Earl et al. 2025), sometimes evolving on week-to-month timescales as the circularization process proceeds. The short-lived, dynamically young disks formed in TDEs differ in formation and evolution timescales from long-term AGN accretion disks, making them prime subjects for investigating disk-formation physics. Radiative-transfer and disk-wind models developed for AGN (e.g., H. M. L. G. Flohic et al. 2012; S. Y.

Yong et al. 2017) demonstrate that outflows, optical-depth effects, and non-axisymmetric velocity fields can substantially reshape Balmer-line profiles. Similar physical ingredients are increasingly invoked in TDEs, where emerging debris disks and reprocessing layers introduce comparable kinematic complexity (e.g., T. Hung et al. 2020; T. W. S. Holoien et al. 2019; T. Wevers et al. 2022).

This body of work collectively demonstrates that disk emission is likely widespread in both AGN and TDEs, even if it is only visible under certain geometric or accretion-state conditions. At the same time, it shows that robust, quantitative comparison of disk structures across these populations requires modeling frameworks capable of capturing the full range of plausible disk geometries while remaining computationally tractable for large samples. Motivated by these needs, we introduce FEADME, the Fast Elliptical Accretion Disk Modeling Engine, a GPU-accelerated Python package built using JAX and NumPyro. FEADME implements the complete relativistic elliptical disk model originally developed by M. Eracleous et al. (1995), with optional additional components to allow for simultaneous fitting of spectral profiles. By leveraging JAX’s just-in-time compilation, vectorized likelihood evaluation, and NumPyro’s efficient No-U-Turn Sampler (NUTS) and Stochastic Variational Inference (SVI) inference engines, FEADME achieves order-of-magnitude speedups compared to previous disk-modeling frameworks, making rigorous Bayesian inference feasible for large AGN and TDE samples.

In this paper, we apply FEADME to two complementary data sets. The first is a sample of 237 AGN DPEs identified and continuum-subtracted by C. Ward et al. (2024). The second is a sample of five TDEs with clear disk-like Balmer emission, chosen for their well-resolved double-peaked or highly asymmetric  $H\alpha$  profiles. Our aim is to test whether the disk parameters derived from AGN and TDEs occupy distinct or overlapping regions of parameter space, and whether these distributions reflect differences in their disk-formation mechanisms. If TDE disks truly arise from the rapid circularization of stellar debris, they may exhibit distinct geometric or kinematic signatures compared to the long-lived, secularly evolving disks of AGN. Conversely, if their inferred disk structures are similar, it would suggest that the physics of line formation in relativistic disks is remarkably universal across transient and persistent accretion episodes.

This paper is organized as follows: in Section 2, we describe the AGN and TDE samples used in this study and summarize the spectral preparation procedures. Section 3 details the FEADME modeling framework, including the elliptical disk implementation, sampling strategy, and model selection approach. In Section 4, we present the fitting procedure and outline the three model families evaluated for each spectrum. Section 5 reports the

inferred disk parameters for the AGN and TDE samples, along with a clustering analysis of the AGN population. In Section 6, we compare the global distributions of disk parameters between AGN and TDEs and discuss their physical implications. We summarize the main conclusions in Section 7.

## 2. DATA AND SAMPLE SELECTION

### 2.1. AGN Sample

The sample of 250 double-peaked emitters (DPEs) used in this analysis was drawn from the catalog of C. Ward et al. (2024), which identified a large sample of broad-line AGN exhibiting double-peaked H $\alpha$  emission profiles. Their classification was based on an accretion disk model analysis, followed by an automated identification pipeline and manual verification.

C. Ward et al. (2024) initially identified 1549 broad-line AGN from the ZTF survey (E. C. Bellm et al. 2019; M. J. Graham et al. 2019; R. Dekany et al. 2020) and modeled their broad H $\alpha$  emission using a circular accretion disk model after subtracting the stellar continuum using the Penalized Pixel Fitting (pPXF) code (M. Cappellari 2023). The accretion disk model, based on the framework of K. Chen et al. (1989), also includes a single spiral arm component which was required to describe the asymmetry in the red and blue shoulders commonly observed in disk emitters (T. Storchi-Bergmann et al. 2003b). Of the 1549 AGN, 1302 yielded successful disk model fits, while the remaining spectra were dominated by narrow-line emission, preventing reliable modeling.

The selection of the DPE sample was carried out based on the accretion disk model parameters and additional spectral features of the broad H $\alpha$  line. Their classification required that the emission feature show a clear dip or plateau between the two peaks of the broad H $\alpha$  line, a velocity offset greater than 500 km s<sup>-1</sup> between the peak of the broad line and the narrow H $\alpha$  component, and a flux ratio asymmetry between the red and blue peaks exceeding 0.8. The quantitative cuts were followed up by a visual inspection to refine the selection. After these refinements, the final sample contained 250 DPEs, accounting for 19.2% of the strongly variable broad-line AGN sample. The remaining 1052 AGN without prominent double-peaked profiles comprised the control sample.

Following C. Ward et al. (2024), we use the continuum-subtracted spectra<sup>4</sup> for all objects in their DPE catalog as input to our FEADME modeling. Although the catalog contains 250 DPEs, our final sample includes 237 AGN; thirteen spectra were excluded because the corresponding data products were missing or malformed. Removing these cases ensures that all retained spectra have consistent formatting and sufficient data quality for reliable modeling.

### 2.2. TDE Sample

We select five TDEs from the literature that have exhibited double-peaked emission in their Balmer line profiles: AT 2018hyz, AT 2018zr, AT 2020nov, AT 2020zso, and PTF09djl. Representative spectra for each event are shown in Figure 1. All TDE spectra were retrieved from the WISEREP archive<sup>5</sup> except for AT 2020zso, whose reduced spectra were provided through private communication.

AT 2018hyz displayed prominent double-peaked H $\alpha$  emission, with two distinct peaks separated by 9320 km/s (T. Hung et al. 2020; P. Short et al. 2020). This feature was most pronounced about 51 days after discovery and could be well-modeled by a low eccentricity ( $e \approx 0.1$ ) accretion disk. AT 2018zr (PS18kh) showed flat-topped and double-peaked H $\alpha$  profiles that evolved over time, though some debate exists over whether these originated from a disk or an outflow (T. W. S. Holoin et al. 2019). AT 2020nov showed unique characteristics that blurs the line between TDE and AGN accretion disks. The double-peaked emission lines, particularly prominent in the Balmer series, are well-modeled by an elliptical accretion disk characterized by moderate eccentricity ( $e \approx 0.53$ ) and inclination ( $i \approx 49$  deg), with an unusually large outer radius of  $\sim 5 \times 10^4 R_g$  (N. Earl et al. 2025). AT 2020zso demonstrated clear double-peaked profiles in multiple emission lines including H $\alpha$  and He II (T. Wevers et al. 2022). Its H $\alpha$  profile was well-fit by a highly inclined ( $i \approx 85$  deg), highly elliptical ( $e \approx 0.97$ ) accretion disk. PTF09djl exhibited an unusual H $\alpha$  profile with two peaks separated by  $3.5 \times 10^4$  km/s, which was interpreted as emission from a highly elliptical ( $e \approx 0.97$ ) and inclined ( $i \approx 88$  deg) disk (F. K. Liu et al. 2017). However, the low signal-to-noise of these spectra has led to some uncertainty in this interpretation.

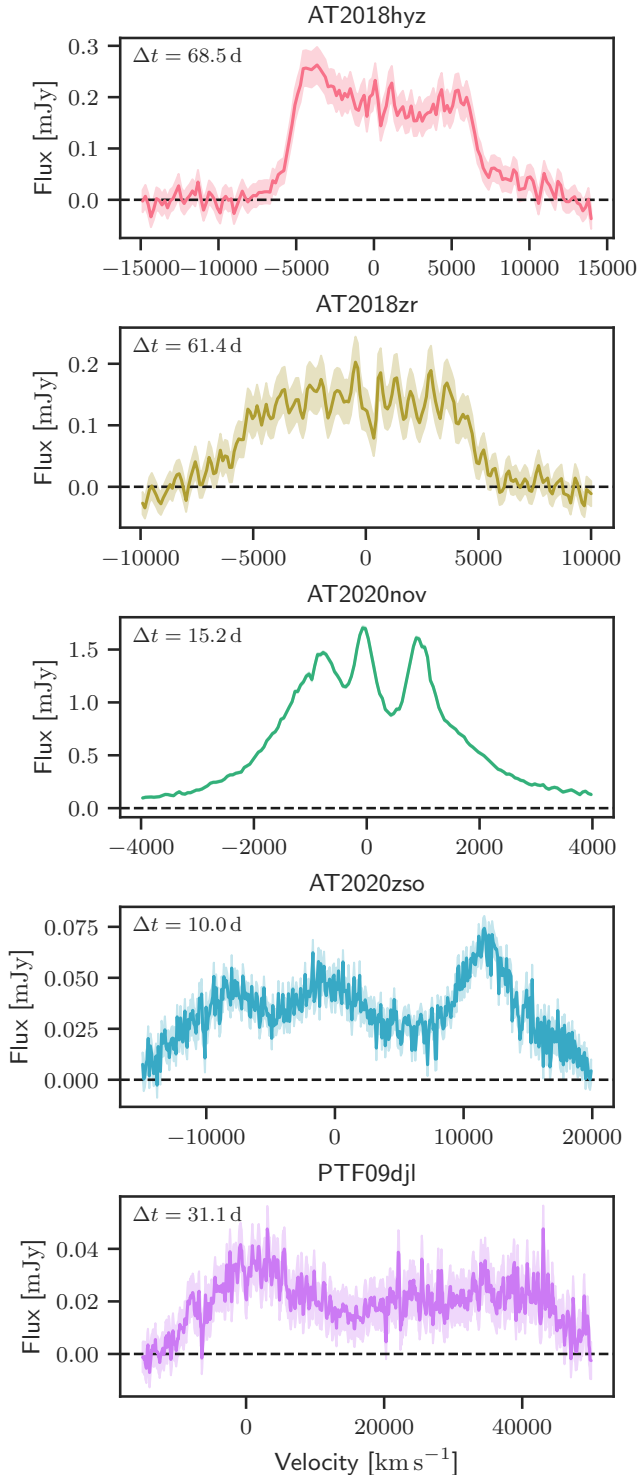
Other TDEs that have been fit with accretion disk models were also considered, most notably ASSASN-14li. Although ASASSN-14li has been modeled successfully with an extremely eccentric, nearly face-on disk geometry (R. Cao et al. 2018), its observed Balmer-line properties are single-peaked and only moderately broad, lacking the distinct double-peaked or strongly asymmetric morphology that motivates our disk-emitter selection. Because our goal is to compare systems with spectroscopically evident disk-like Balmer emission rather than sources whose disk interpretations relies primarily on model-dependent reconstruction, ASASSN-14li does not meet our selection criteria and is therefore not included in our sample.

## 3. THE FEADME MODELING FRAMEWORK

FEADME (Fast Elliptical Accretion Disk Modeling Engine) is an open-source Python package for probabilistic

<sup>4</sup> <https://github.com/charlotteaward/ZTF-DPEs>

<sup>5</sup> <https://www.wiserep.org>



**Figure 1.** Representative rest-frame spectra for each of the five TDEs in our sample (AT 2018hyz, AT 2018zr, AT 2020nov, AT 2020zso, and PTF09djl) showing broad, double-peaked  $H\alpha$  emission consistent with disk-like kinematics. Each panel displays a single epoch selected to highlight the double-peaked structure, with the offset from peak optical brightness shown in the top-left corner. The diversity in peak separation, asymmetry, and profile shape reflects underlying differences in disk geometry, orientation, and emission structure, which we explore in detail through elliptical disk modeling in Section 5.3.

modeling of broad emission-line profiles in astrophysical spectra. The package combines a modern Bayesian inference workflow with high-performance computation provided by `JAX` and `NumPyro`, and offers a flexible, modular interface for configuring samplers, priors, and model parameters. While the present work focuses on modeling accretion-disk emission using the elliptical disk model of *M. Eracleous et al. (1995)*, `FEADME` is structured as a general tool for spectroscopic modeling, with an inference pipeline designed to accommodate additional sampling methods and modeling components as the package evolves. The software is publicly available on GitHub.<sup>6</sup>

### 3.1. Architecture and Package Design

The `FEADME` architecture separates the physical model definition from the inference workflow, enabling a clean interface for specifying model parameters, prior distributions, likelihood functions, and sampling configurations. This design allows users to adjust or extend the inference pipeline (i.e. the choice of sampler, initialization method, etc.) without modifying the underlying physical model implementation. The package currently provides built-in support for both Stochastic Variational Inference (SVI) and Hamiltonian Monte Carlo (HMC), and is structured so that future sampling algorithms within the `NumPyro` ecosystem can be integrated with minimal overhead.

### 3.2. Elliptical Accretion Disk Model

`FEADME` implements the relativistic elliptical accretion disk model developed by *M. Eracleous et al. (1995)* and adopted for large samples of disk emitters by *I. V. Strateva et al. (2003)*. In this framework, the broad Balmer emission arises from gas in a geometrically thin, optically thick disk, with material following coplanar Keplerian ellipses. A single set of global parameters describes the disk geometry and kinematics, and the observed double-peaked line profile is obtained by integrating the locally emitted specific intensity over the entire disk surface, including relativistic Doppler and gravitational effects.

We follow the notation of *I. V. Strateva et al. (2003)* and parameterize radii in units of the gravitational radius,  $R_g$ . The disk is specified by an inner and outer semimajor axis,  $\xi_1$  and  $\xi_2$ , expressed in units of  $R_g$ ; an eccentricity  $e$  common to all orbits; an inclination angle  $i$  between the disk normal and the observer’s line of sight; and an azimuthal orientation angle  $\phi_0$  giving the direction of apocenter in the disk plane. The line emissivity per unit area is taken to follow a power-law in radius,  $\epsilon(\xi) \propto \xi^{-q}$ , where  $q$  is a free parameter. Local line broadening is described by a Gaussian with velocity dispersion  $\sigma$ , which encodes non-Keplerian motions (e.g., turbulence or vertical structure) in the line-emitting gas.

<sup>6</sup> <https://github.com/nmearl/feadme>

We introduce the dimensionless frequency shift

$$X \equiv \frac{\nu}{\nu_0} - 1, \quad (1)$$

where  $\nu_0$  is the rest-frame line frequency. The observed, frequency-resolved line flux can then be written as

$$F_X = \frac{M^2 \nu_0 \cos i}{d^2} \int_{\xi_1}^{\xi_2} \int_0^{2\pi} \xi I_{\nu_e}(\xi, X) \times D^3(\xi, \phi) \Psi(\xi, \phi) d\phi d\xi, \quad (2)$$

where  $M$  is the black hole mass,  $d$  is the luminosity distance,  $D$  is the relativistic Doppler factor, and  $\Psi(\xi, \phi)$  is a weak-field light-bending correction (see [M. Eracleous et al. 1995](#); [I. V. Strateva et al. 2003](#) for the full expressions). The integral is taken over radius (expressed as the semimajor axis  $\xi$ ) and azimuthal angle  $\phi$  in the disk plane.

In the weak-field limit appropriate for  $\xi \gtrsim 100 R_g$ , the light-bending factor  $\Psi(\xi, \phi)$  can be expressed as

$$\Psi(\xi, \phi) = 1 + \xi^{-1} \left( \frac{1 - \sin i \cos \phi}{1 + \sin i \cos \phi} \right), \quad (3)$$

and the impact parameter  $b/r$  that enters the relativistic transfer function satisfies

$$\frac{b}{r} \approx \sqrt{1 - \sin^2 i \cos^2 \phi} \Psi(\xi, \phi). \quad (4)$$

The locally emitted specific intensity at frequency  $\nu_e$  (in the emitter's frame) is modeled as a Gaussian profile whose normalization follows the radial emissivity law:

$$I_{\nu_e}(\xi, X) = \frac{\epsilon_0 \xi^{-q}}{4\pi\sqrt{2\pi}\sigma} \exp\left[-\frac{(1+X-D)^2\nu_0^2}{2\sigma^2 D^2}\right], \quad (5)$$

where  $\epsilon_0$  is a constant emissivity normalization and  $\sigma$  is the local broadening in frequency units.

The elliptical geometry is introduced through the relation between the semimajor axis  $\xi$  and the instantaneous radius of an orbit at azimuth  $\phi$ :

$$\xi(\phi) = \frac{\xi(1+e)}{1-e\cos(\phi-\phi_0)}, \quad (6)$$

where  $e$  is the common eccentricity and  $\phi_0$  specifies the orientation of apocenter. The relativistic Doppler factor  $D(\xi, \phi)$  and Lorentz factor  $\gamma(\xi, \phi)$  are computed following the weak-field expressions in [M. Eracleous et al. \(1995\)](#) and [I. V. Strateva et al. \(2003\)](#), which include special relativistic Doppler shifts and boosting, transverse Doppler effects, and gravitational redshift. In the notation of those works,  $D^{-1}$  is a function of  $(\xi, e, \phi, \phi_0, i)$  and the impact parameter  $b/r$ , and  $\gamma$  depends on the local orbital speed along the elliptical trajectory.

In practice, **FEADME** evaluates Equation 2 on an observed-frame wavelength grid, using the mapping

$$X = \frac{\nu}{\nu_0} - 1 = \frac{\lambda_0}{\lambda} - 1, \quad (7)$$

and transforming  $F_X$  into a model line profile  $F_\lambda$  on the user-specified wavelength sampling. This wavelength-space formulation allows for direct masking of contaminating features and straightforward addition of auxiliary components (e.g., narrow emission lines or an extra broad Gaussian) in the same spectral frame. It is important to note, however, that **FEADME** does not evaluate the absolute flux predicted by Equation 2. The multiplicative prefactor, which includes the dependence on black hole mass, distance, and the unknown absolute emissivity normalization, is factored out and replaced by a single fitted amplitude parameter. This approach preserves the full kinematic and relativistic structure of the disk model while allowing the data to determine the overall line flux. Because the black hole masses in our AGN and TDE samples are not known with sufficient precision for absolute flux calibration, **FEADME** focuses on modeling the normalized line profile shape rather than predicting its absolute flux.

The full elliptical disk model as implemented in **FEADME** is thus controlled by a set of seven free parameters,

$$\{q, \xi_1, \xi_2, i, \sigma, \phi_0, e\}, \quad (8)$$

which together determine the radial extent, orbital shape, orientation, emissivity gradient, and local broadening of the disk. These parameters fully specify the forward model that produces the broad H $\alpha$  disk component used throughout this work.

The disk model is implemented in **JAX** as a fully differentiable forward model. Automatic differentiation enables efficient gradient-based sampling, while **JAX**'s vectorization and just-in-time compilation substantially reduce the computational cost of evaluating the line profile across large parameter grids or during repeated likelihood evaluations.

### 3.3. *JAX and NumPyro for Probabilistic Inference*

**FEADME** integrates tightly with **NumPyro** ([D. Phan et al. 2019](#); [E. Bingham et al. 2018](#)), a probabilistic programming library that provides state-of-the-art Bayesian inference methods including SVI and HMC via the No-U-Turn Sampler (NUTS). Because **NumPyro** is built on **JAX**, all inference routines benefit from automatic differentiation, XLA compilation, and optional GPU acceleration.

The package currently supports sampling using either SVI or Hamiltonian Monte Carlo (HMC), via NUTS. SVI allows for using flexible normalizing-flow guides (e.g., **AutoBNFNormal**) that learn expressive approximations to the posterior and provide high-quality initialization points for subsequent samplers, while NUTS sampling is used for generating asymptotically exact

posterior samples using gradient information from the differentiable disk model.

The sampler interface is intentionally extensible: users may configure sampler parameters, chain counts, initialization strategies, and inference diagnostics, and future versions of the package may incorporate additional inference backends.

### 3.4. *Command-Line Interface and Reproducible Workflows*

To facilitate large-scale modeling and reproducible analyses, **FEADME** includes a command-line interface (CLI) that allows users to run model fits, specify sampler options, define priors, and export posterior samples without writing custom Python scripts. The CLI supports both single-spectrum and batch workflow execution, making it suitable for interactive use, survey-scale pipelines, and deployment on HPC or cloud computing platforms. Example CLI usage is provided in Appendix A.

## 4. MODEL FITTING PROCEDURE

To characterize the physical properties of the broad-line emission in both the AGN and TDE samples, we fit every spectrum with the **FEADME** elliptical accretion disk framework described above. Each fit is carried out in **NumPyro** using a two-stage inference strategy: we first train a variational guide with Neural Transport (NeuTra; M. Hoffman et al. 2019) reparameterization, and then draw the final posterior samples with the NUTS operating in the learned latent space. For the variational stage we adopt the **AutoBNFNormal** guide, which provides a flexible normalizing-flow approximation to the posterior and learns an invertible transport map that flattens correlations among the physical parameters. The resulting transport map and variational mean/covariance are then used to initialize NUTS in a well-adapted region of parameter space, improving mixing and effective sample sizes for the subsequent MCMC.

For each object (both AGN and TDE), we construct and fit three physically motivated model families, each representing a different hypothesis for the origin of the broad H $\alpha$  emission:

1. **Disk + Narrow Lines + Broad Gaussian (Full Model)**: an elliptical accretion disk component, Gaussian narrow-line components for the H $\alpha$  complex (H $\alpha$ , [N II]  $\lambda\lambda 6548, 6584$ , and [S II]  $\lambda\lambda 6716, 6731$  where present), and an additional broad Gaussian centered on H $\alpha$  to capture non-disk high-velocity emission such as BLR contributions or winds.
2. **Disk + Narrow Lines Only (No-BLR Model)**: the same elliptical disk and narrow-line components as above, but without the broad Gaussian. This model tests whether a disk plus

narrow emission alone can reproduce the observed profile.

3. **Broad Gaussian + Narrow Lines Only (No-Disk Model)**: a broad H $\alpha$  Gaussian together with the narrow-line components, but no disk. This provides a BLR-like baseline model against which we can assess whether disk kinematics are statistically required.

For the AGN sample, all narrow-line components are modeled directly. For the TDE sample, host-galaxy narrow-line emission was removed as part of the respective prior analyses and published reductions, whenever a high-quality host reference spectrum was available. This host subtraction was possible for all TDEs in our sample except AT 2020nov, whose narrow lines are therefore retained and modeled explicitly in the same manner as for the AGN. Apart from this external preprocessing step, all TDE spectra are fit with the same three model families as the AGN.

Each model family is sampled with NeuTra-initialized NUTS in **NumPyro**. For every object–model combination, we run two independent chains, monitor the Gelman–Rubin convergence diagnostic ( $\hat{R}$ ), and require  $\hat{R} < 1.01$  for all free parameters. Typical fits require a few thousand post-warmup samples per chain to achieve convergence. Posterior samples are transformed back into physical space via the learned transport map, yielding constraints on the seven elliptical disk parameters (inner radius  $\xi_1$ , outer radius  $\xi_2$ , eccentricity  $e$ , inclination  $i$ , apocenter angle  $\phi_0$ , emissivity index  $q$ , and turbulent broadening  $\sigma$ ), together with the properties of any additional emission line components (e.g., velocity widths of Gaussian narrow- or broad-line components) when included.

To determine which of the three model families best describes each spectrum, we perform model comparison using approximate leave-one-out (LOO) cross-validation as implemented in the **ArviZ compare** function. For each object we pass the three corresponding posterior ensembles to **compare**, compute the LOO information criterion for all three models, and adopt the model with the lowest (best) score as the preferred physical description. Across the 237 AGN in our sample, the majority (161; 67.9%) favor the ‘Full Model’, indicating that most AGN require both a disk component and an additional broad H $\alpha$  Gaussian. Another 48 AGN (20.3%) prefer the ‘No-BLR Model’, consistent with a pure disk interpretation without additional broad emission, while 28 AGN (11.8%) are best described by the ‘No-Disk Model’, suggesting that a non-disk broad component provides a better explanation of their line profiles. The TDEs show a qualitatively similar but more pronounced behavior. For the TDEs, we removed two late-time AT 2018hyz spectra ( $\Delta t = 189.5$  d and 217.5 d) due to poor data quality, yielding a final sample of 29 usable epochs. Of these, 23 (79.3%) favor the ‘Full Model’, 4 (13.8%) pre-

fer the ‘No-Disk Model’, and only 2 (6.9%) select the ‘No-BLR Model’. This distribution indicates that TDE double-peaked Balmer emission often includes a substantial non-disk component in addition to any disk-like structure. In all subsequent analyses we therefore use only the posterior samples associated with each object’s preferred LOO-selected model family rather than enforcing a single fixed model choice across the full sample.

## 5. RESULTS

### 5.1. Disk Parameters Across AGN Sample

Across the full AGN sample, the elliptical disk fits reveal a population of broad-line emitters with broadly similar global geometries but substantial diversity in detailed structure. Representative examples of these fits, illustrating the decomposition into disk and non-disk components, are shown in Figure 2. The combined parameter distributions for all AGN, summarized in the final row of Table 1 and shown as grey violins in Figure 3, are characteristic of the classical disk-emitter population identified in previous work (e.g. M. Eracleous et al. 1995; I. V. Strateva et al. 2003; C. Ward et al. 2024). The typical disk is moderately to highly inclined, with a median inclination of  $i = 61.1^{+8.1}_{-20.3}$  deg, with errorbars in this section representing the 68th percentile ranges in the fitted parameter distributions, consistent with the expectation that strong double-peaked profiles preferentially arise in systems that are not viewed face-on. The inner and outer radii cluster around  $\xi_1 = 2225^{+2326}_{-1475} R_g$  and  $\xi_2 = 11882^{+14938}_{-7138} R_g$ , respectively, with long tails toward larger radii. These scales are consistent with a Balmer-emitting layer in the outer disk rather than emission from near the innermost stable circular orbit.

The orbits within these disks are typically non-circular. The median eccentricity of the global AGN sample is  $e = 0.64^{+0.28}_{-0.38}$ , indicating that a large fraction of sources require appreciable ellipticity to account for their asymmetric shoulders and unequal peak heights. These large ellipticities are consistent with other work that has used large-scale asymmetric features such as spiral arms or precessing disks to explain the double peaked structure in systems such as Arp 102B and 3C 390.3 (e.g. T. Storchi-Bergmann et al. 2003a; J. S. Schimoia et al. 2012, 2017), and suggests that similar kinematic configurations are common across the broader DPE population. The apocenter orientation angle,  $\phi_0$ , spans nearly the full 0 deg to 360 deg range, with a median of  $\phi_0 = 220.1^{+79.1}_{-157.1}$  deg, consistent with an effectively random distribution of elliptical major axes in the disk plane and no preferred apsidal alignment across the sample.

The radial emissivity and local broadening parameters further highlight the physical diversity of the AGN disks. The emissivity index has a median value of  $q = 1.69^{+1.36}_{-0.62}$ , with the wide range indicating that some systems are dominated by emission from the inner disk,

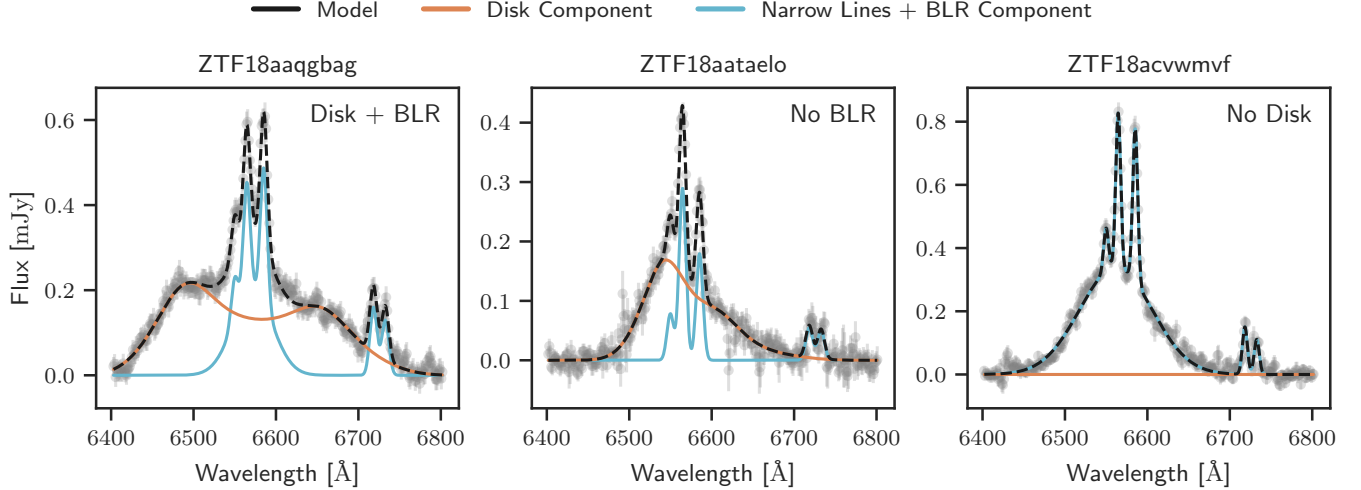
while others exhibit relatively shallow emissivity gradients in which the outer regions contribute significantly to the line flux. This spread in  $q$  is qualitatively consistent with expectations from radiative transfer calculations in disk atmospheres and winds, where the balance between local heating, optical depth, and ionization can shift the dominant line-emitting radius over time (e.g. H. M. L. G. Flohic et al. 2012; S. Y. Yong et al. 2017). The local Gaussian broadening parameter has a median of  $\sigma = 818.7^{+975.8}_{-407.9}$  km s<sup>-1</sup> and exhibits a broad tail toward high values, suggestive of non-Keplerian motions such as turbulence, vertical structure, or small-scale flows within the emitting layer. Similar combinations of ordered rotation and disordered motions have been inferred from detailed modeling of individual broad-line AGN, particularly in systems where disk-wind components interact with the dense disk atmosphere (e.g. E. Bon et al. 2009; W. Kollatschny & M. Zetzl 2013).

Finally, the broad Gaussian component that is included when preferred by the LOO model comparison has a characteristic FWHM of  $\text{FWHM}_{\text{broad}} = 6711^{+3560}_{-3370}$  km s<sup>-1</sup>. This velocity scale is comparable to classical BLR widths and indicates that many AGN require an additional, roughly centrally peaked broad component in addition to the thin elliptical disk in order to reproduce their H $\alpha$  profiles. Taken together, the global parameter distributions show that AGN DPEs occupy a continuous and extended region of disk-parameter space rather than clustering around a single canonical configuration. The disks are typically extended, moderately inclined, and frequently eccentric, with a wide range of emissivity slopes and local broadening, and they are often accompanied by a broader BLR-like or wind-like component. This picture is in line with earlier interpretations of complex broad-line morphologies in AGN as the combined outcome of disk rotation, eccentric structure, and outflows (e.g. M. Eracleous 1998; E. Bon et al. 2009; H. M. L. G. Flohic et al. 2012; C. Ward et al. 2024), and it provides the baseline against which we compare both the cluster-specific archetypes and the TDE disk parameters in the following sections.

### 5.2. Clustering Analysis

We take the curated sample of 237 DPEs from C. Ward et al. (2024) as a starting point for our elliptical accretion disk analysis and comparison, however we observed that many of the spectra in the final sample do not, in practice, display the strong, double-peaked morphology commonly associated with emission from a relativistic accretion disk. Instead, a significant subset of sources exhibit broad emission features that are either shallow, highly asymmetric, or heavily blended with other components. In some cases, the broad profile appears to be better described by a single-peaked Lorentzian or a weakly asymmetric Gaussian profile.

To investigate whether the AGN in our sample occupy distinct regions of disk-parameter space or instead



**Figure 2.** Representative examples of the three model families used to fit the AGN spectra in this work. Each panel shows an observed AGN spectrum (grey), together with the corresponding best-fit model (black dashed line). The contribution from the elliptical accretion disk is shown in gold, while all non-disk line-emission components (narrow lines and the broad Gaussian) are shown in cyan. These examples illustrate the qualitative behavior of the disk+BLR, no BLR, and no disk model families (see Section 4) and highlight the different ways in which disk and non-disk components combine to reproduce the observed H $\alpha$  line profiles.

**Table 1.** Fitted AGN disk parameters by cluster.

ID	$n$	$e$	$i$ [deg]	$\xi_1$ [R $_g$ ]	$\xi_2$ [R $_g$ ]	$q$	$\sigma$ [km s $^{-1}$ ]	$\phi_0$ (deg) [deg]	FWHM [km s $^{-1}$ ]
C0	82	$0.62^{+0.31}_{-0.39}$	$64.6^{+6.4}_{-5.9}$	$1747^{+1458}_{-986}$	$10948^{+8975}_{-5431}$	$1.45^{+0.29}_{-0.49}$	$1160^{+917}_{-530}$	$247.5^{+51.2}_{-85.5}$	$5155.2^{+7170.1}_{-2725.4}$
C1	24	$0.84^{+0.10}_{-0.32}$	$55.6^{+8.5}_{-8.5}$	$3567^{+2793}_{-1984}$	$24275^{+11154}_{-13320}$	$1.91^{+0.36}_{-0.81}$	$379.2^{+367.6}_{-132.5}$	$125.6^{+181.6}_{-77.5}$	$6340.1^{+1681.0}_{-2057.0}$
C2	28	$0.59^{+0.27}_{-0.35}$	$60.7^{+4.2}_{-5.8}$	$3234^{+2070}_{-1421}$	$12010^{+9987}_{-6784}$	$3.45^{+0.30}_{-0.40}$	$962.6^{+832.0}_{-476.4}$	$199.3^{+85.7}_{-116.2}$	$6770.0^{+5313.0}_{-4163.7}$
C3	33	$0.82^{+0.12}_{-0.29}$	$29.1^{+8.9}_{-12.1}$	$1550^{+2909}_{-1173}$	$21114^{+14513}_{-18402}$	$2.13^{+1.07}_{-0.79}$	$636.1^{+765.9}_{-233.2}$	$158.7^{+104.0}_{-114.7}$	$6940.3^{+2295.2}_{-3331.5}$
All	199	$0.64^{+0.28}_{-0.38}$	$61.1^{+8.1}_{-20.3}$	$2225^{+2326}_{-1475}$	$11882^{+14938}_{-7138}$	$1.69^{+1.36}_{-0.62}$	$818.7^{+975.8}_{-407.9}$	$220.1^{+79.1}_{-157.1}$	$6710.6^{+3559.5}_{-3366.5}$

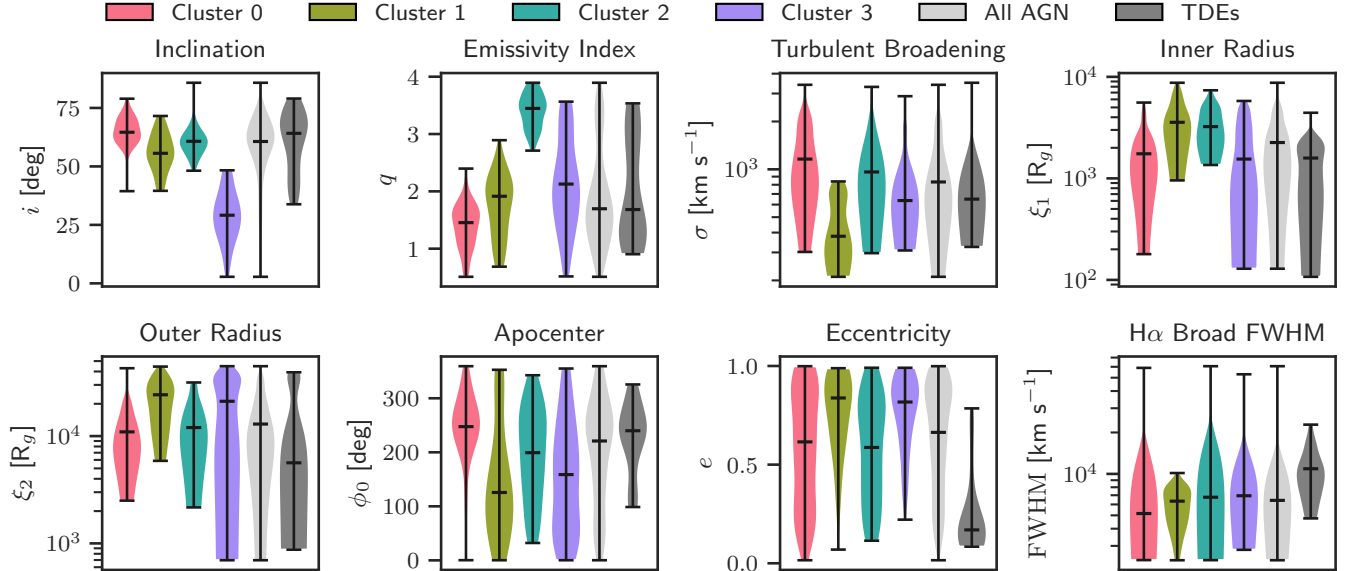
form a continuous family of accretion-disk configurations, we perform an unsupervised clustering analysis using the posterior medians of the LOO-selected best-fit disk model parameters for each source. Our aim is not to identify rigid “AGN classes,” but rather to determine whether the AGN population exhibits any natural concentration points, or archetypal configurations, within the multidimensional disk-parameter manifold.

Before clustering, we first scale the parameters using the `RobustScaler` transformation implemented in `scikit-learn`. This approach centers each feature on its median and scales it according to its interquartile range, thereby reducing the influence of outliers and long-tailed parameter distributions that are common in disk-model fits.

We next assess whether the AGN population exhibits any statistically meaningful cluster structure using the Hopkins statistic (B. Hopkins & J. G. Skellam 1954). The Hopkins statistic measures the spatial randomness

of a dataset by comparing the nearest-neighbor distances of uniformly sampled synthetic points to those of the real data. Values near  $H \approx 0.5$  indicate a distribution consistent with complete spatial randomness, values near  $H \approx 0$  indicate regular (ordered or grid-like) structure, and values near  $H \approx 1$  indicate strong clustering. For our AGN disk-parameter vectors, we obtain a Hopkins statistic of  $H = 0.663$ , suggesting only mild clustering tendency and implying that the AGN are not cleanly partitioned into discrete groups. Instead, they appear to lie on a smooth manifold with localized density enhancements.

To further explore the geometric structure of this manifold, we perform a dimension reduction using the UMAP algorithm (L. McInnes et al. 2018), which is well-suited for revealing latent structure in nonlinear data. We project the parameter space into a five-component UMAP embedding using `n_neighbors = 15`, which balances local and global structure in the manifold approxi-



**Figure 3.** Distribution of elliptical disk model parameters across AGN clusters identified via UMAP + HDBSCAN clustering (see Section 5.2), as well as for the full AGN sample (light grey) and the full TDE sample (dark grey). Each panel corresponds to one of the eight model parameters: the seven elliptical disk parameters (inclination ( $i$ ), emissivity index ( $q$ ), turbulent broadening ( $\sigma$ ), inner radius ( $\xi_1$ ), outer radius ( $\xi_2$ ), apocenter angle ( $\phi_0$ ), and eccentricity ( $e$ )), along with the FWHM of the broad Gaussian component included when preferred by the fit. Widths indicate the relative density of posterior samples aggregated across all objects in a group, while the horizontal black bar shows the medians of the distributions. The TDE sample exhibits systematically lower eccentricities and outer radii compared to the AGN cluster populations. Differences in parameter distributions highlight the diversity of disk structures present in both persistent (AGN) and transient (TDE) accretion flows.

mation. The resulting embedding preserves the continuity of the parameter space while enhancing the visibility of density concentrations that may correspond to physical archetypes of disk structure.

We then apply HDBSCAN, a hierarchical density-based clustering algorithm (R. J. G. B. Campello et al. 2013), to the five-dimensional UMAP space. HDBSCAN naturally identifies clusters of varying density and labels low-density points as noise, making it particularly well-suited to astrophysical datasets that may not adhere to rigid spherical cluster shapes. Using a minimum cluster size of 15 and a minimum samples parameter of 5, HDBSCAN identifies four statistically significant clusters. A total of 167 AGN (83.9%) are assigned to one of these clusters, while 32 AGN (16.1%) fall below the clustering threshold and remain unlabeled. We interpret these unlabeled sources not as a separate category, but as transitional or intermediate objects within the broader disk-parameter manifold.

The clustering analysis reveals that the AGN do not separate into discrete classes. Instead, they occupy a continuous, smooth manifold with four regions of locally enhanced density that we interpret as archetypal configurations of elliptical accretion disk structure. These archetypes represent common geometric or dynamical configurations rather than sharply bounded classes, and

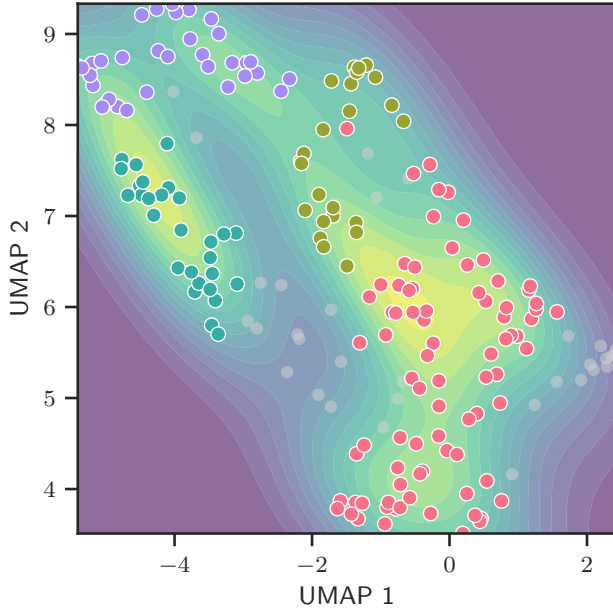
the transitional objects underscore the continuity of the accretion-disk parameter space.

### 5.3. TDE Disk Structure

The TDEs in our sample exhibit substantial diversity in their inferred disk geometries and kinematics, reflecting the range of conditions under which debris circularizes and begins emitting Balmer-line radiation. The full set of fitted parameters for all epochs is provided in Table 2, and the examples of the temporal evolution for each event are shown in Figure 6.

A defining feature of the TDE disks is the wide range of eccentricities recovered across epochs. Although many spectra favor low eccentricity ( $e \sim 0.1$  to  $0.2$ ), consistent with the formation of relatively circular disks once debris undergoes self-intersection and shock dissipation (T. Hung et al. 2020; J. Krolik et al. 2016), several early epochs, particularly in AT 2018hyz and AT 2018zr, show elevated eccentricities. This behavior aligns with expectations that TDE disks begin in a highly noncircular state and evolve toward more circular configurations as circularization proceeds.

The inferred inclinations cluster between  $i \sim 55$  deg to  $75$  deg, indicating that the Balmer emission is generally produced in moderately to highly inclined viewing geometries. These inclinations are consistent with the broad, double-peaked, or asymmetric struc-

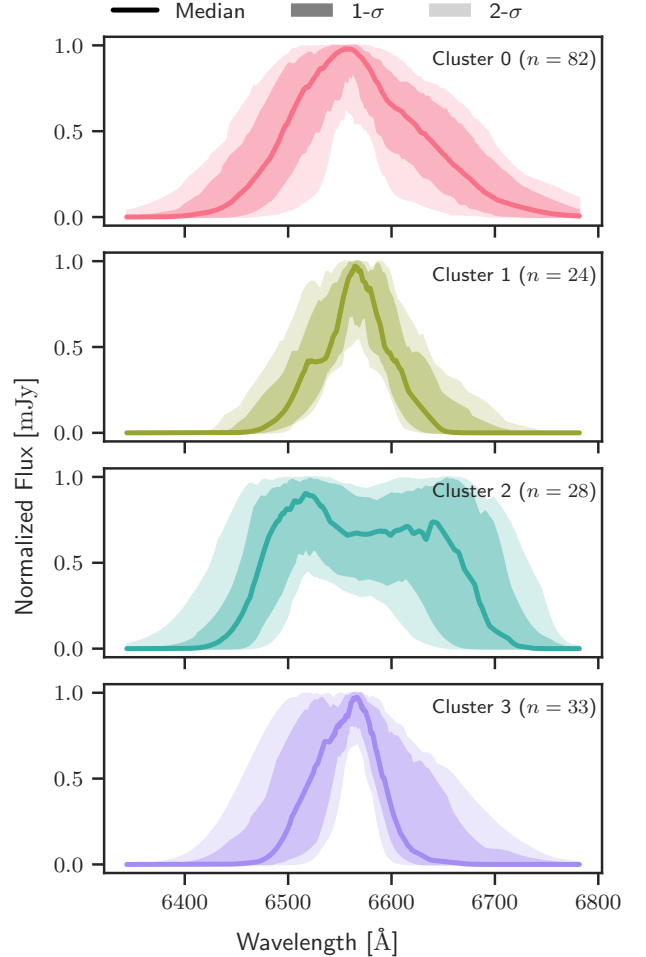


**Figure 4.** Two-dimensional projection of the 5-component UMAP embedding of the AGN disk parameter space. Colored regions indicate high-density areas associated with the four HDBScan-identified AGN clusters, while individual spectra not assigned to any cluster (i.e. transitional or ambiguous sources) are shown as grey points. The density shading reflects the concentration of objects in the reduced space, with clearer structure visible in well-populated clusters. This projection highlights the dominant morphological groups and the presence of a significant population of spectra that occupy intermediate or low-density regions of the disk parameter manifold.

tures observed in the line profiles of our TDE sample, all of which require larger projected rotational velocities to reproduce their observed peak separations.

The radial scales of the line-emitting regions also span a broad range. The median inner radius across all TDE epochs is  $\xi_1 \approx 1600 R_g$ , with individual events occasionally reaching substantially smaller radii when the line-emitting region extends deeper into the gravitational potential. Outer radii show even greater diversity. Many epochs favor compact disks with  $\xi_2 \sim 3 \times 10^3 R_g$  to  $9 \times 10^3 R_g$ , indicative of line formation in the inner, newly assumed debris disk. In contrast, AT 2020nov exhibits unusually large outer radii up to  $\sim 3 \times 10^4 R_g$ , consistent with models suggesting that its Balmer emission arose from a larger, pre-existing disk illuminated during the flare (N. Earl et al. 2025).

The radial emissivity gradients likewise reflect varied disk conditions. The emissivity index spans  $q \approx 1$  to 3.5, with many epochs favoring steep profiles in which the innermost radii dominate the Balmer emission. Such steep emissivities are consistent with radiative-transfer



**Figure 5.** Median rest-frame H $\alpha$  fitted disk profiles for each of the four AGN clusters identified via UMAP + HDBScan. Solid lines represent the median spectrum within each cluster, while the shaded regions denote the 16th–84th percentile range (dark shading) and the 5th–95th percentile range (light shading) at each wavelength. Spectra were normalized and interpolated onto a common grid prior to stacking. The profiles illustrate the diversity in line shape and structure across the clusters, with some groups showing well-separated double peaks and others exhibiting more blended or asymmetric broad-line features.

calculations of young TDE disks, where heating and ionization are concentrated near the circularization radius and optical depths can strongly weight the inner regions (H. M. L. G. Flohic et al. 2012). The local turbulent broadening parameter shows substantial spread, with median  $\sigma \approx 650 \text{ km s}^{-1}$ , but individual epochs reaching well above  $1000 \text{ km s}^{-1}$ . This variation may reflect differences in the vertical structure and turbulence of the line-emitting layer, as suggested by studies that decompose broad-line widths into rotational and turbulent components and relate the latter to the disk scale height

(e.g. W. Kollatschny et al. 2012; W. Kollatschny & M. Zetzl 2013), as well as additional broadening from shocks and highly disturbed flows during debris circularization and disk assembly in TDEs (e.g. H. Shiokawa et al. 2015; T. Piran et al. 2015; T. W. S. Holoien et al. 2019).

In addition to the elliptical disk, most TDE spectra require a broad Gaussian component with characteristic FWHM  $\sim 4 \times 10^3 \text{ km s}^{-1}$  to  $12 \times 10^3 \text{ km s}^{-1}$ . This necessity is consistent with previous interpretations of TDE Balmer-line formation, where disk emission is accompanied by contributions from reprocessing layers, outflowing gas, or high-velocity debris streams (e.g. T. W. S. Holoien et al. 2019; P. Short et al. 2020). The amplitude and width of this component often evolve rapidly with time, paralleling changes in continuum luminosity and debris dynamics.

## 6. DISCUSSION

### 6.1. Comparing Disk Structure in TDEs and AGN

Figure 7 compares the global disk-parameter distributions of the TDE sample to those of the AGN disk emitters modeled in this work. Despite the distinct physical origins of steady-state AGN accretion disks and newly assembled TDE debris flows, we find that the majority of structural parameters inferred from the line profile fits occupy broadly overlapping regions of parameter space. Mann–Whitney U tests indicate that for five of the seven parameters, the TDE distributions are statistically indistinguishable from the full AGN sample at  $p > 0.1$ , indicating that the global disk morphology in TDEs is not strongly segregated from that of classical AGN double-peaked emitters.

The most significant difference arises in the eccentricity distribution. TDE disks have a median  $e = 0.17$ , substantially lower than the AGN median  $e = 0.64$ , and the difference is highly significant ( $p = 2.3 \times 10^{-8}$ ). One possible interpretation is that many TDE disks have already undergone substantial circularization by the time broad Balmer emission emerges. Recent hydrodynamic studies show that interactions between returning debris stream and the nascent disk, either through stream-disk collisions, pericenter shocks, or mixing at the nozzle, can reduce orbital eccentricity and promote disk formation (E. Steinberg & N. C. Stone 2024; X. Huang et al. 2024). This picture is consistent with low eccentricities inferred for AT 2018hyz and PS18kh (T. Hung et al. 2020; T. W. S. Holoien et al. 2019, respectively). However, we note that not all simulations predict rapid circularization; some scenarios involving inefficient shocks or delayed inflow can maintain highly eccentric debris for extended periods. By contrast, the persistence of the large  $e$  in many AGN is commonly attributed to long-lived non-axisymmetric structure, such as spiral arms or precessing elliptical disks, that can be maintained over secular timescales (M. Eracleous et al. 1995; I. V. Strateva et al. 2003; T. Storchi-Bergmann et al. 2017). The separation in eccentricity therefore likely reflects differ-

ences in dynamical history and dissipation mechanisms rather than a fundamental distinction in the instantaneous geometry of the line-emitting regions.

In contrast, the inclination distributions show no significant difference between TDEs and the full AGN sample ( $p = 0.15$ ). Both populations cluster around  $i \approx 55\text{deg} - 65\text{deg}$ , consistent with the long-standing interpretation that double-peaked profiles require moderately high inclinations to produce large projected rotational velocities (K. Chen et al. 1989; M. Eracleous & J. P. Halpern 2003). Individual cluster comparisons reveal one notable outlier: AGN Cluster 3 shows significantly lower inclinations ( $p = 6.3 \times 10^{-9}$ ), consistent with its observed weaker peak asymmetries. The similarity between TDE and AGN inclination distributions suggests that orientation, not formation channel, primarily determines whether a disk produces an observable double-peaked emitter.

The emissivity index  $q$  also shows broad overlap between TDEs and AGN. The TDE median ( $q \approx 1.68$ ) is nearly identical to that of the full AGN sample ( $p = 0.83$ ), although differences appear when comparing with individual clusters: C0 and C2 show statistically significant deviations. These variations reflect the diversity of disk conditions such as optical depth, temperature gradients, and radiative-transfer effects, that shape the radial weighting of Balmer emission. Theoretical models of disk atmospheres and disk winds (H. M. L. G. Flohic et al. 2012; S. Y. Yong et al. 2017) predict a wide range of effective emissivity slopes, and observational modeling of AGN BLR profiles likewise suggests no universal value (e.g. E. Bon et al. 2009; W. Kollatschny & M. Zetzl 2013). Our results reinforce this view that TDEs and AGN both occupy a continuum of  $q$  values, indicating that the physics regulating line formation (e.g., local heating and ionization) is similar in both environments.

Local velocity broadening shows a comparable behavior. TDEs have median  $\sigma \approx 650 \text{ km s}^{-1}$ , while AGN exhibit slightly higher values ( $\sigma \approx 820 \text{ km s}^{-1}$ ), but the difference is not statistically significant ( $p = 0.11$ ). Cluster-by-cluster comparisons reveal that TDEs differ significantly from C0 and C1 but not from C2 or C3. Because  $\sigma$  encodes sub-Keplerian motions, it provides a useful probe of local disk dynamics. The agreement between TDEs and AGN supports the idea that once a disk is emitting Balmer lines, the microphysical broadening mechanisms are similar regardless of whether the disk is transient or long-lived.

The radial scales of the line-emitting regions show more nuanced behavior. TDEs generally favor more compact outer radii than AGN clusters C1 or C3, but their distribution is statistically indistinguishable from the full AGN sample ( $p = 0.11$ ). The median TDE  $\xi_2 \sim 5600 R_g$  lies squarely in the range inferred for classical AGN disk emitters (e.g. M. Eracleous et al. 1995; I. V. Strateva et al. 2003; C. Ward et al. 2024). A subset of TDEs (most dramatically AT 2020nov) show extremely

**Table 2.** Fitted disk parameters for TDE observations.

TDE	Offset [d]	$e$	$i$ [deg]	$\xi_1$ [ $R_g$ ]	$\xi_2$ [ $R_g$ ]	$q$	$\sigma$ [ $\text{km s}^{-1}$ ]	$\phi_0$ (deg) [deg]	FWHM [ $\text{km s}^{-1}$ ]
AT2018hyz	10	$0.09^{+0.09}_{-0.05}$	$57.5^{+18.9}_{-17.0}$	$472.7^{+180.6}_{-189.8}$	$5636^{+12144}_{-3228}$	$3.03^{+0.34}_{-0.32}$	$713.7^{+602.6}_{-304.9}$	$248.7^{+73.0}_{-43.1}$	$11178.9^{+1609.7}_{-1472.7}$
	27	$0.20^{+0.10}_{-0.06}$	$72.6^{+13.2}_{-16.6}$	$1046^{+165}_{-259}$	$9360^{+19278}_{-6120}$	$3.49^{+0.33}_{-0.35}$	$554.2^{+422.6}_{-184.5}$	$116.7^{+25.5}_{-34.1}$	$11237.3^{+670.2}_{-655.5}$
	40	$0.61^{+0.17}_{-0.16}$	$66.1^{+13.2}_{-14.8}$	$2404^{+542}_{-599}$	$3623^{+1184}_{-988}$	$3.37^{+0.43}_{-0.42}$	$597.9^{+205.0}_{-175.3}$	$220.8^{+7.4}_{-6.7}$	$10271.9^{+593.6}_{-524.3}$
	56	$0.13^{+0.05}_{-0.02}$	$66.1^{+13.5}_{-13.7}$	$1977^{+420}_{-466}$	$2826^{+650}_{-695}$	$1.68^{+0.49}_{-0.41}$	$860.9^{+136.8}_{-125.7}$	$260.3^{+32.4}_{-28.1}$	$10903.0^{+1159.2}_{-1183.7}$
	68	$0.14^{+0.03}_{-0.02}$	$79.0^{+6.9}_{-6.9}$	$1981^{+241}_{-233}$	$3062^{+364}_{-327}$	$1.22^{+0.28}_{-0.15}$	$659.3^{+119.6}_{-121.8}$	$315.3^{+10.4}_{-16.8}$	$11307.4^{+924.2}_{-795.2}$
	80	$0.10^{+0.03}_{-0.02}$	$76.1^{+9.2}_{-8.6}$	$1898^{+195}_{-207}$	$3256^{+366}_{-307}$	$1.36^{+0.48}_{-0.25}$	$496.8^{+131.0}_{-110.9}$	$304.2^{+17.7}_{-26.8}$	$10331.2^{+534.3}_{-566.2}$
	97	$0.21^{+0.07}_{-0.05}$	$63.7^{+14.2}_{-13.6}$	$762.9^{+210.0}_{-203.3}$	$3169^{+1229}_{-844}$	$2.05^{+0.43}_{-0.46}$	$818.6^{+315.0}_{-282.7}$	$263.0^{+24.6}_{-22.4}$	$7533.3^{+1811.3}_{-1821.2}$
	111	$0.13^{+0.03}_{-0.02}$	$74.3^{+9.8}_{-8.7}$	$1586^{+255}_{-257}$	$3090^{+361}_{-300}$	$1.27^{+0.39}_{-0.20}$	$517.8^{+174.4}_{-142.0}$	$283.0^{+29.6}_{-36.0}$	$9995.4^{+395.9}_{-379.9}$
	131	$0.16^{+0.06}_{-0.04}$	$68.9^{+11.7}_{-14.2}$	$1712^{+257}_{-393}$	$2606^{+427}_{-565}$	$1.75^{+0.76}_{-0.45}$	$440.8^{+136.0}_{-96.5}$	$242.2^{+20.3}_{-14.9}$	$8737.7^{+393.1}_{-377.6}$
AT2018zr	26	$0.49^{+0.27}_{-0.18}$	$64.1^{+17.8}_{-18.5}$	$2241^{+664}_{-833}$	$5389^{+6836}_{-2028}$	$3.41^{+0.40}_{-0.48}$	$635.3^{+328.7}_{-213.3}$	$117.0^{+12.9}_{-16.3}$	$16979.9^{+1200.6}_{-925.2}$
	40	$0.28^{+0.21}_{-0.12}$	$68.0^{+14.7}_{-19.1}$	$2223^{+721}_{-784}$	$7115^{+10895}_{-3215}$	$2.77^{+0.66}_{-0.62}$	$1343^{+637}_{-536}$	$106.8^{+31.3}_{-43.6}$	$22800.8^{+2750.0}_{-2127.3}$
	52	$0.17^{+0.10}_{-0.06}$	$64.1^{+16.3}_{-18.7}$	$1770^{+1223}_{-836}$	$5411^{+2158}_{-1871}$	$0.90^{+0.41}_{-0.26}$	$1369^{+307}_{-263}$	$98.7^{+36.4}_{-39.4}$	$15235.6^{+1116.7}_{-950.2}$
	61	$0.18^{+0.13}_{-0.09}$	$68.6^{+13.4}_{-15.4}$	$2410^{+876}_{-781}$	$7145^{+8455}_{-3079}$	$2.72^{+0.70}_{-0.98}$	$933.7^{+551.5}_{-406.0}$	$101.4^{+50.2}_{-63.9}$	$16143.9^{+139.7}_{-1156.1}$
AT2020nov	-23	$0.14^{+0.07}_{-0.04}$	$53.3^{+6.7}_{-6.0}$	$4429^{+848}_{-857}$	$39286^{+5415}_{-5116}$	$1.35^{+0.14}_{-0.15}$	$368.4^{+39.0}_{-33.5}$	$236.5^{+21.7}_{-17.8}$	$7990.8^{+836.1}_{-697.9}$
	-9	$0.43^{+0.04}_{-0.06}$	$55.7^{+3.2}_{-3.6}$	$257.2^{+188.5}_{-91.3}$	$38643^{+2245}_{-2151}$	$1.10^{+0.04}_{-0.04}$	$423.4^{+27.9}_{-26.8}$	$204.6^{+3.2}_{-2.6}$	$11588.7^{+1287.3}_{-1063.4}$
	15	$0.11^{+0.03}_{-0.03}$	$45.9^{+5.2}_{-4.3}$	$2273^{+367}_{-292}$	$28479^{+4479}_{-3855}$	$1.20^{+0.06}_{-0.05}$	$325.0^{+24.8}_{-17.2}$	$323.0^{+8.7}_{-15.5}$	$9064.3^{+214.0}_{-211.9}$
	41	$0.12^{+0.04}_{-0.03}$	$47.6^{+6.1}_{-5.3}$	$907.3^{+587.4}_{-448.7}$	$38684^{+4950}_{-4780}$	$1.10^{+0.17}_{-0.19}$	$401.7^{+40.4}_{-38.1}$	$258.0^{+28.0}_{-26.1}$	$7377.2^{+755.8}_{-645.5}$
	156	$0.23^{+0.16}_{-0.10}$	$42.8^{+9.9}_{-9.1}$	$1133^{+966}_{-988}$	$34331^{+9267}_{-9988}$	$1.25^{+0.18}_{-0.19}$	$632.2^{+128.5}_{-115.9}$	$229.8^{+27.7}_{-18.8}$	--
	163	$0.15^{+0.05}_{-0.03}$	$36.5^{+1.9}_{-1.5}$	$688.1^{+65.7}_{-56.2}$	$34651^{+2758}_{-2267}$	$1.22^{+0.03}_{-0.03}$	$649.4^{+19.1}_{-20.2}$	$239.9^{+16.5}_{-15.4}$	$4752.6^{+62.9}_{-31.5}$
	268	$0.08^{+0.10}_{-0.05}$	$33.8^{+6.7}_{-6.4}$	$2214^{+1438}_{-902}$	$34012^{+9641}_{-10187}$	$1.21^{+0.32}_{-0.33}$	$431.4^{+128.6}_{-88.5}$	$233.2^{+83.3}_{-47.0}$	$5914.2^{+3402.8}_{-900.7}$
	271	$0.14^{+0.20}_{-0.08}$	$40.5^{+8.5}_{-7.8}$	$510.4^{+765.1}_{-301.1}$	$36739^{+7883}_{-8977}$	$1.22^{+0.19}_{-0.22}$	$682.7^{+159.8}_{-157.1}$	$242.1^{+92.3}_{-44.5}$	--
AT2020zso	-2	$0.17^{+0.12}_{-0.09}$	$73.9^{+7.1}_{-8.5}$	$345.7^{+35.3}_{-43.8}$	$1554^{+423}_{-251}$	$2.88^{+0.35}_{-0.35}$	$1052^{+569}_{-420}$	$152.6^{+11.1}_{-22.4}$	$22420.5^{+530.2}_{-571.6}$
	10	$0.73^{+0.08}_{-0.08}$	$66.5^{+8.7}_{-7.4}$	$475.7^{+52.1}_{-55.4}$	$873.7^{+128.8}_{-127.3}$	$3.47^{+0.27}_{-0.24}$	$2394^{+167}_{-199}$	$208.3^{+2.1}_{-2.3}$	$4968.9^{+363.9}_{-323.8}$
PTF09djl	2	$0.75^{+0.07}_{-0.05}$	$73.6^{+3.9}_{-5.5}$	$107.3^{+6.0}_{-4.2}$	$1201^{+5653}_{-564}$	$3.53^{+0.15}_{-0.22}$	$3505^{+520}_{-540}$	$325.6^{+4.1}_{-4.7}$	$15494.5^{+914.7}_{-822.5}$
	31	$0.79^{+0.04}_{-0.04}$	$39.2^{+6.3}_{-3.9}$	$121.3^{+23.3}_{-14.1}$	$24005^{+12031}_{-8012}$	$2.62^{+0.04}_{-0.04}$	$972.3^{+350.3}_{-306.2}$	$260.6^{+5.6}_{-5.9}$	$18194.1^{+717.7}_{-657.7}$
All		$0.17^{+0.34}_{-0.06}$	$64.1^{+9.5}_{-21.7}$	$1586^{+660}_{-1133}$	$5636^{+29349}_{-2845}$	$1.68^{+1.69}_{-0.47}$	$649.4^{+448.9}_{-219.3}$	$239.9^{+46.4}_{-123.0}$	$10903.0^{+5642.2}_{-3444.7}$

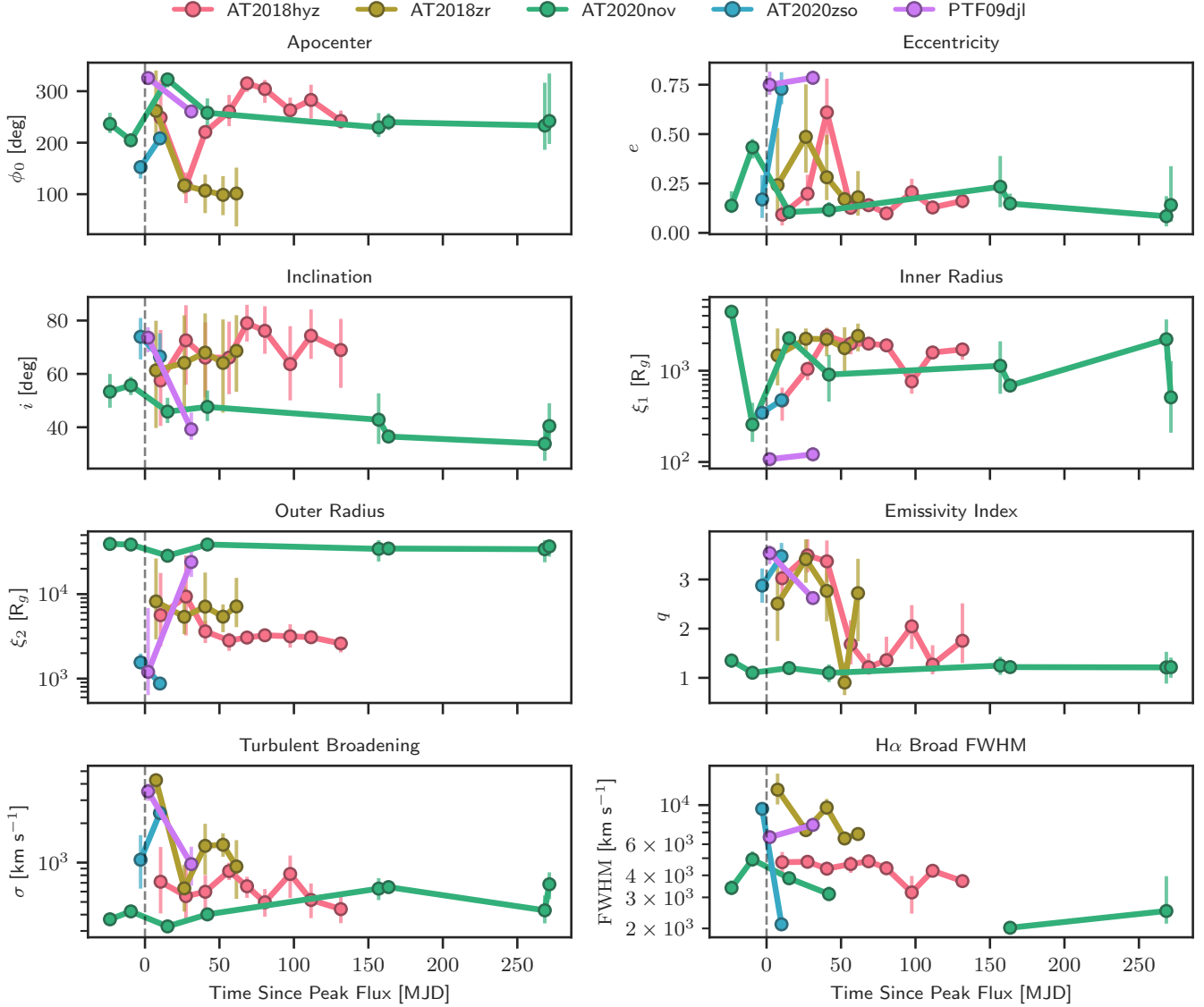
extended outer radii approaching  $\sim 3 \times 10^4 R_g$ , consistent with spectroscopic modeling indicating a large pre-existing disk that participates in reprocessing the TDE flare (N. Earl et al. 2025). Such events demonstrate that TDEs can form or illuminate disk structures comparable in extent to AGN disks, particularly when a quiescent disk is already present. On the other hand, several TDEs such as AT 2020zso and AT 2018hyz exhibit relatively compact outer radii, far smaller than those inferred for the AGN disks. Such small characteristic scales are consistent with the expectation that these systems may harbor genuinely newly formed disks whose line-emitting regions have not yet expanded beyond the immediate circularization zone.

Inner radii show similar overlap. Although TDEs yield statistically smaller  $\xi_1$  on average than the full AGN distribution ( $p = 2.1 \times 10^{-3}$ ), the values themselves are well within the AGN range. The modest shift toward smaller radii may simply reflect the deeper gravitational

potential reached by circularizing debris compared to the outer AGN disk zones where Balmer lines typically form.

The apocenter angles ( $\phi_0$ ) show no significant differences for the full samples, reinforcing the idea that azimuthal asymmetry is stochastic rather than population-specific. Cluster-level deviations, notably with C1 and C3, may reflect either the observational effects or strong non-axisymmetric structure in those AGN groups (e.g., precessing elliptical disks or spiral arms), consistent with theoretical models of long-lived disk perturbations (e.g. T. Storchi-Bergmann et al. 2017; C. Ward et al. 2024).

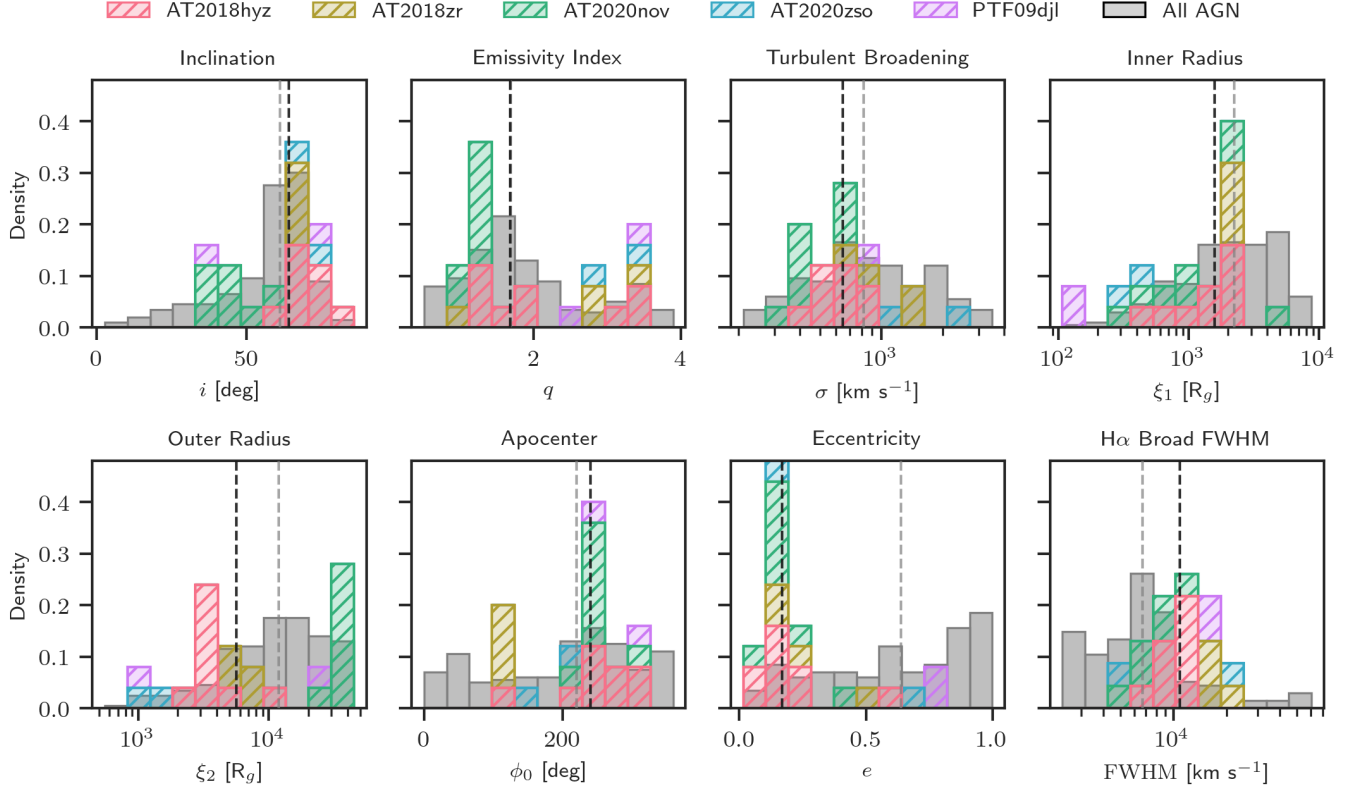
A further distinction between the two populations emerges in the FWHM of the additional broad H $\alpha$  component included when preferred by model selection. While both AGN and TDEs frequently require this non-disk emission component, the TDEs exhibit systematically broader broad-line widths, with a median  $\text{FWHM}_{\text{TDE}} \approx 1.1 \times 10^4 \text{ km s}^{-1}$ , compared to



**Figure 6.** Time evolution of the fitted elliptical disk model parameters for the five TDEs in our sample. Each panel shows one of the eight parameters (the seven disk parameters along with the FWHM of the broad Gaussian component when included). Time is normalized individually for each TDE, with  $t = 0$  corresponding to the epoch of maximum observed brightness. Points represent the posterior medians at each epoch, with error bars indicating the 16th–84th percentile credible intervals. The evolving parameter values reflect changes in the line-emitting region and disk structure over time.

$\text{FWHM}_{\text{AGN}} \approx 6.7 \times 10^3 \text{ km s}^{-1}$  for the AGN sample. A Mann-Whitney U-test confirms that the two distributions are statistically distinct ( $p = 4.18 \times 10^{-5}$ ), indicating that the velocity structure of this non-disk component differs between the transient and persistent accretion environments. The TDE widths are fully consistent with optical TDE samples, which typically show hydrogen and helium lines with characteristic FWHM of order  $10^4 \text{ km s}^{-1}$  (P. Charalampopoulos et al. 2022; E. J. Parkinson et al. 2022), where large AGN surveys generally find broad H $\alpha$  widths of a few  $\sim 10^3 \text{ km s}^{-1}$  for classical broad-line regions (A. Shapovalova et al.

2009; J. Negus et al. 2024). In this context, the AGN broad Gaussian likely traces a more virialized BLR or disk-wind component, while the broader TDE feature reflects a dynamically younger and more extreme environment in which high-velocity gas is associated with shocks, reprocessing layers, or partially unbound debris. The difference in broad-line FWHM therefore reinforces the view that, although both populations require an extra high-velocity emitter beyond the disk, the kinematic state of this component is systematically more energetic in TDEs than in the steady AGN hosts.



**Figure 7.** Comparison of posterior parameter distributions between the AGN and TDE samples. Each panel shows one of the eight fitted parameters (the seven elliptical disk parameters, plus the FWHM of the broad Gaussian component, when included). Grey histograms represent the stacked posterior distributions for all AGN in the sample. Overlaid colored histograms show the stacked TDE posterior distributions, shaded by individual event. Vertical dashed lines mark the median of the full AGN distribution (grey) and the combined TDE distribution (black). The comparison highlights that most structural parameters overlap substantially between the two populations, while eccentricity provides the strongest evidence for differing disk-formation pathways.

Taken together, these comparisons indicate that, with the exception of eccentricity, TDE disks are not strongly distinguishable from AGN disks in the parameter spaces probed by Balmer-line modeling. Both populations occupy overlapping ranges of inclination, emissivity slope, local broadening, and radial extent. The only robust separation matches theoretical expectations of rapid circularization in TDE debris streams and the long-lived eccentric modes in AGN disks. In all other respects, TDEs that exhibit double-peaked or asymmetric Balmer emission appear to trace the same family of relativistic, rotationally supported line-emitting structures studied in AGN.

This similarity supports a unified interpretation in which the geometry and kinematics of the line-emitting disk, rather than its long-term accretion history, dominate the emergent line profiles. For TDEs, this implies that once debris settles into a rotating configuration capable of producing double-peaked emission, its spectroscopic appearance becomes largely indistinguishable

from that of an AGN disk, offering a natural explanation for the close overlap in Figure 7.

### 6.2. Limitations of the Disk Model

A key assumption of the Eracleous elliptical disk model, and therefore of our FEADME fits, is that the line-emitting gas resides in a geometrically thin, optically thick accretion disk whose kinematics are dominated by nearly Keplerian orbital motion. This approximation is well motivated for sub-Eddington AGN disks, where standard thin-disk solutions are expected to apply at the radii that produce the broad Balmer emission lines. Indeed, *F. K. Liu et al. (2017)* explicitly interpret the eccentric disk they fit to PTF09djl as an optically thick, geometrically thin configuration at a sub-Eddington accretion rate, and more generally find that Balmer line emission can be produced in the illuminated surface layers of such disks.

For TDEs, however, this assumption is more tentative. A large fraction of optically selected TDEs appear to radiate near or above their Eddington limit at early times, implying that their accretion flows are likely to

be geometrically thick and to launch powerful, optically thick winds rather than remaining a razor-thin configuration (L. E. Strubbe & E. Quataert 2009, 2011; B. D. Metzger & N. C. Stone 2016). In AT 2018hyz, for example, T. Hung et al. (2020) infer an Eddington ratio of  $\sim 0.6$  at peak and argue that radiation pressure should puff up the disk and drive an optically thick outflow. Similar super-Eddington phases and associated reprocessing layers are generically expected in TDEs from theory and simulations, which predict quasi-spherical or funnel-shaped envelopes and strong disk winds that can obscure or reshape the underlying thin disk (K. Ohsuga & S. Mineshige 2011; J. C. McKinney et al. 2015; A. Sądowski et al. 2016; L. Dai et al. 2018; C. Bonnerot & W. Lu 2020). In this regime, the Balmer-emitting region may trace the photosphere of a thickened disk or the base of a wind rather than the midplane of a thin disk.

Our successful fits to the double-peaked Balmer lines in TDEs therefore do not guarantee that the underlying flow is literally a geometrically thin disk. Instead, the Eracleous model should be viewed as an effective parameterization of the projected velocity field in the line-forming region. The fitted inner and outer radii, emissivity index, and inclination encode where and how the  $H\alpha$  photons are produced and escape, but the true geometry may involve a vertically extended disk atmosphere, a thick torus, or a combination of disk and outflow. This is particularly relevant for events with high inferred Eddington ratios and strong evidence for reprocessing envelopes, where our “disk radii” likely trace the characteristic orbital scales of the emitting material rather than sharp boundaries in a razor-thin structure.

These caveats are less severe for the AGN sample, where the accretion flows may be closer to the classical thin-disk regime and where the same Eracleous formalism has been shown to reproduce the line profiles of long-lived double-peaked emitters (M. Eracleous et al. 1995; I. V. Strateva et al. 2003). Nevertheless, even in AGN there is strong evidence that disk winds and vertically extended broad-line regions can modify Balmer-line profiles and blur the distinction between thin-disk and wind-dominated geometries. For both AGN and TDEs, our FEADME parameters should therefore be interpreted as characterizing the kinematics and radial scales of the line-emitting gas under the thin-disk approximation, with the understanding that deviations from geometric thinness and the presence of optically thick outflows may systematically bias the mapping between these parameters and the true three-dimensional structure of the accretion flow.

## 7. CONCLUSIONS

In this work we introduce FEADME, a new GPU-accelerated Python framework for modeling broad emission-line profiles using the relativistic elliptical accretion disk model of M. Eracleous et al. (1995). By cou-

pling a fully differentiable forward model with modern Bayesian inference tools provided by JAX and NumPyro, FEADME enables efficient, large-sample modeling of disk-dominated Balmer-line emission in both AGN and TDEs. The flexibility and performance of this framework make it possible to conduct statistical rigorous, population-level comparisons of accretion-disk structure with uniform methodology.

We applied FEADME to 237 AGN from the DPE sample of C. Ward et al. (2024) and to a sample of five TDEs known to exhibit disk-like Balmer emission. For every object, we evaluated three physically motivated model families and selected the optimal one using approximate leave-one-out cross-validation. The resulting posterior samples allowed us to reconstruct the distributions of disk geometry, kinematics, emissivity structure, and local broadening across both populations.

Our main findings are as follows:

1. The AGN sample shows a diverse range of eccentricity, inclination, emissivity slope, radial extent, and local broadening. Rather than forming discrete classes, AGN disks populate a continuous manifold with localized regions of high density. This complexity likely reflects a combination of secular disk evolution, non-axisymmetric perturbations, and diverse radiative environments.
2. The majority of TDE parameter distributions are statistically indistinguishable from those of the AGN as a whole. The most significant difference is in eccentricity where TDE disks are consistently more circular, consistent with theoretical expectations that debris streams rapidly dissipate eccentricity during circularization. Aside from this distinction, TDEs occupy the same geometric and kinematic regime as classical AGN disk emitters.
3. Model selection indicates that a combination of disk emission and a broad Gaussian component provides the best description for the majority of both populations. However, the kinematic properties of this broad component differ: TDEs exhibit systematically larger FWHM values than AGN, suggesting that the non-disk emission in TDEs arises from dynamically young, shock- or outflow-dominated gas rather than the more virialized broad-line region typically present in AGN.

Taken together, these results suggest a unified physical picture for broad Balmer-line emission in galactic nuclei. Once a rotating disk forms and becomes the dominant line-emitting region, through either long-term accretion in AGN or rapid debris circularization in TDEs, its spectral appearance is governed primarily by the same relativistic and radiative processes. The close overlap between TDE and AGN disk parameters underscores this continuity and demonstrates that the diversity of

double-peaked and asymmetric Balmer profiles can be explained through a common geometric framework.

FEADME provides a scalable platform for future spectroscopic studies of accretion disks across cosmic time, from AGN variability surveys to time-domain studies of nuclear transients. As larger samples of TDEs and disk-emitting AGN become available from ZTF, LSST, and forthcoming spectroscopic follow-up programs, the ability to model disks with consistent physical assumptions will enable more robust analysis into the formation, evolution, and dynamics of accretion flows around supermassive black holes.

## APPENDIX

### A. FEADME INTERFACE AND TEMPLATE SYSTEM

To facilitate reproducibility and large-scale fitting, FEADME provides a command-line interface (CLI) with dedicated subcommands for different inference backends. All workflows share two primary inputs: (1) a spectral data file containing wavelength, flux, and flux uncertainty columns, and (2) a JSON template file that defines the physical model, parameter priors, and masking configuration.

The spectral data are supplied as an ASCII CSV file with three columns `wave` (in Å), `flux` (in mJy), and `flux_err` (in mJy), which are read in and stored in a `Data` object. The observed-frame wavelength grid is used directly, with an optional internal rebinning step available for coarse-velocity binning. The accompanying template file is parsed into a `Template` object, which specifies the redshift, white-noise hyperparameter, masking intervals, and the full set of disk and line profiles to be modeled.

#### A.1. CLI Entry Points and Sampler Configuration

The top-level CLI group `feadme` currently exposes two subcommands:

- `feadme nuts`: run Hamiltonian Monte Carlo using the No-U-Turn Sampler (NUTS);
- `feadme svi`: run Stochastic Variational Inference (SVI) with a normalizing-flow guide.

Both subcommands accept the same core arguments (a template path, a data path, and an output directory) with additional options specific to the chosen sampler. A typical NUTS invocation for a single spectrum is:

```
1 feadme nuts \
2   --template-path templates/ztf18aaaotwe.json \
3   --data-path data/ZTF18aaaotwe.csv \
4   --output-path results/ZTF18aaaotwe \
5   --num-warmup 1000 \
6   --num-samples 3000 \
7   --num-chains 2 \
8   --target-accept-prob 0.8 \
9   --max-tree-depth 10 \
```

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```
10 --dense-mass \
11 --neutra
```

**Listing 1.** CLI example for FEADME NUTS invocation

This command loads the template and data, optionally performs an experimental least-squares pre-fit to initialize the priors, constructs the `NumPyro` model, and runs NUTS with the requested configuration. Results are written to `results.nc` in the specified output directory, together with summary tables and diagnostic plots. If a `results.nc` file already exists, FEADME will load the existing posterior instead of re-running the sampler.

The SVI workflow is analogous:

```
1 feadme svi \
2   --template-path templates/ztf18aaaotwe.json \
3   --data-path data/ZTF18aaaotwe.csv \
4   --output-path results/ZTF18aaaotwe_svi \
5   --num-steps 25000 \
6   --num-posterior-samples 2000 \
7   --learning-rate 1e-3 \
8   --decay-rate 0.1 \
9   --decay-steps 20000 \
10  --hidden-factors 8 --hidden-factors 8 \
11  --num-flows 2
```

**Listing 2.** CLI example for FEADME SVI invocation

Here, the user controls the number of optimization steps, the number of posterior samples drawn from the converged guide, and the architecture of the normalizing-flow guide (via the `hidden_factors` and `num_flows` options). In both modes, sampler settings are collected into a `Config` object (either `NUTSSamplerSettings` or `SVISamplerSettings`), and passed together with the `Template` and `Data` to the appropriate sampler class.

#### A.2. Template Structure

The JSON template defines the physical and statistical structure of the model in a fully declarative way. At the top level, a template contains:

- `"name"`: an identifier for the object;
- `"obs_date"`: an optional observation timestamp (used for time-series analyses);

- "redshift": a parameter block describing the redshift prior;
- "white\_noise": a parameter block for the log-amplitude of additional white noise;
- "disk\_profiles": a list of elliptical disk components (typically one per broad Balmer line);
- "line\_profiles": a list of Gaussian line components (narrow and broad);
- "mask": a list of wavelength intervals to include in the fit.

Each parameter block (for example, "redshift" or "sigma" in the disk profile) is itself a small dictionary with a common schema: `distribution` (e.g., "uniform", "log\_uniform", "normal", etc.), `low`, `high`, `loc`, `scale`, a Boolean `fixed` flag, and an optional `shared` field that links parameters across components. The `circular` flag indicates angular parameters (such as the apocenter angle) for which circular statistics are used internally.

The example below shows a shortened template used in this work to fit a double-peaked H $\alpha$  profile with an elliptical disk plus narrow and broad Gaussian components, as well as the neighboring [N II]  $\lambda\lambda 6548, 6584$  and [S II]  $\lambda\lambda 6716, 6731$  narrow lines:

```

1 {
2   "name": "ZTF18aaaotwe",
3   "obs_date": 0.0,
4   "redshift": {
5     "distribution": "uniform",
6     "fixed": false,
7     "low": 0.06533,
8     "high": 0.06686,
9     "loc": 0.06609,
10    "scale": 0.0132
11  },
12  "white_noise": {
13    "distribution": "uniform",
14    "fixed": false,
15    "low": -10,
16    "high": 1,
17    "loc": -7,
18    "scale": 1
19  },
20
21  "disk_profiles": [
22    {
23      "name": "halpha_disk",
24      "center": {
25        "distribution": "uniform",
26        "fixed": true,
27        "low": 6557.819,
28        "high": 6567.819,
29        "value": 6562.819

```

```

30    },
31    "inner_radius": {
32      "distribution": "log_uniform",
33      "fixed": false,
34      "low": 100.0,
35      "high": 10000.0,
36      "loc": 1000.0
37    },
38    "radius_scale": {
39      "distribution": "uniform",
40      "low": 0.0,
41      "high": 1.0,
42      "loc": 0.5
43    },
44    "eccentricity": {
45      "distribution": "uniform",
46      "low": 0.0,
47      "high": 1.0,
48      "loc": 0.5
49    },
50    "inclination": {
51      "distribution": "uniform",
52      "low": 0.0,
53      "high": 1.5708, // 0 to pi/2
54      "loc": 0.7854
55    },
56    "apocenter": {
57      "distribution": "uniform",
58      "circular": true,
59      "low": 0.0,
60      "high": 6.2832, // 0 to 2*pi
61      "loc": 3.1416
62    },
63    "q": {
64      "distribution": "uniform",
65      "low": 0.5,
66      "high": 4.0,
67      "loc": 2.5
68    },
69    "sigma": {
70      "distribution": "log_uniform",
71      "low": 200.0,
72      "high": 5000.0,
73      "loc": 1000.0
74    }
75  }
76 ],
77
78  "line_profiles": [
79    {
80      "name": "halpha_narrow",
81      "shape": "gaussian",
82      "center": {
83        "distribution": "uniform",
84        "fixed": true,
85        "low": 6560.819,
86        "high": 6564.819,

```

```

87     "value": 6562.819,
88     "shared": "halpha_disk"
89   },
90   "vel_width": {
91     "distribution": "log_uniform",
92     "low": 10.0,
93     "high": 1000.0,
94     "loc": 500.0
95   },
96   "amplitude": {
97     "distribution": "uniform",
98     "low": 0.0,
99     "high": 0.72,
100    "loc": 0.5
101  }
102 },
103
104 {
105   "name": "halpha_broad",
106   "shape": "gaussian",
107   "center": {
108     "distribution": "uniform",
109     "fixed": false,
110     "low": 6527.819,
111     "high": 6597.819,
112     "loc": 6562.819
113   },
114   "vel_width": {
115     "distribution": "uniform",
116     "low": 1000.0,
117     "high": 6000.0,
118     "loc": 500.0
119   },
120   "amplitude": { ... }
121 },
122
123 // Additional [N II] and [S II]
narrow components omitted for
brevity

```

```

124   ],
125
126   "mask": [
127     { "lower_limit": 6822.997, "
128       upper_limit": 7249.434 }
129   ]

```

**Listing 3.** Excerpt of a FEADME template for ZTF18aaaotwe. In-depth descriptions of parameter definitions can be found on the GitHub repository<sup>6</sup>.

In this example, the H $\alpha$  disk profile is defined by the seven elliptical disk parameters used throughout this work (inclination, inner radius, radius scale, eccentricity, apocenter angle, emissivity index  $q$ , and local broadening  $\sigma$ ), while the narrow and broad Gaussian components share or inherit parameters where appropriate. For instance, the narrow H $\alpha$  center is shared with the disk center, and several of the forbidden narrow lines share the same velocity width as the narrow H $\alpha$  component.

Because the modeling logic is fully encoded in the template, users can construct a wide range of models (pure disks, disk plus broad-line components, or line-only models) without modifying the source code. The same template description is used for both NUTS and SVI runs, enabling consistent comparisons across sampler types and facilitating survey-scale, reproducible modeling of double-peaked emitters.

The full FEADME source code, including the CLI, sampler implementations, template system, and all supporting modules, is publicly available on GitHub<sup>6</sup>. The project is actively developed and welcomes community engagement: users are encouraged to submit issues, request features, open discussions, or contribute enhancements through pull requests. By maintaining FEADME as an open and extensible platform, our goal is to support reproducible analysis workflows and facilitate broader community involvement in the study of disk-emitting AGN and TDEs.

## REFERENCES

- Bellm, E. C., Kulkarni, S. R., Graham, M. J., et al. 2019, *PASP*, 131, 018002, doi: [10.1088/1538-3873/aaecbe](https://doi.org/10.1088/1538-3873/aaecbe)
- Bingham, E., Chen, J. P., Jankowiak, M., et al. 2018, arXiv e-prints, arXiv:1810.09538, doi: [10.48550/arXiv.1810.09538](https://doi.org/10.48550/arXiv.1810.09538)
- Bon, E., Popović, L. Č., Gavrilović, N., La Mura, G., & Mediavilla, E. 2009, *MNRAS*, 400, 924, doi: [10.1111/j.1365-2966.2009.15511.x](https://doi.org/10.1111/j.1365-2966.2009.15511.x)
- Bonnerot, C., & Lu, W. 2020, *MNRAS*, 495, 1374, doi: [10.1093/mnras/staa1246](https://doi.org/10.1093/mnras/staa1246)
- Campello, R. J. G. B., Moulavi, D., & Sander, J. 2013, in *Advances in Knowledge Discovery and Data Mining*, ed. J. Pei, V. S. Tseng, L. Cao, H. Motoda, & G. Xu (Berlin, Heidelberg: Springer Berlin Heidelberg), 160–172
- Cao, R., Liu, F. K., Zhou, Z. Q., Komossa, S., & Ho, L. C. 2018, *MNRAS*, 480, 2929, doi: [10.1093/mnras/sty1997](https://doi.org/10.1093/mnras/sty1997)
- Cappellari, M. 2023, *MNRAS*, 526, 3273, doi: [10.1093/mnras/stad2597](https://doi.org/10.1093/mnras/stad2597)
- Chajet, L. S., & Hall, P. B. 2013, *MNRAS*, 429, 3214, doi: [10.1093/mnras/sts580](https://doi.org/10.1093/mnras/sts580)
- Charalampopoulos, P., Leloudas, G., Malesani, D. B., et al. 2022, *A&A*, 659, A34, doi: [10.1051/0004-6361/202142122](https://doi.org/10.1051/0004-6361/202142122)

- Chen, K., Halpern, J. P., & Filippenko, A. V. 1989, *ApJ*, 339, 742, doi: [10.1086/167332](https://doi.org/10.1086/167332)
- Dai, L., McKinney, J. C., Roth, N., Ramirez-Ruiz, E., & Miller, M. C. 2018, *ApJL*, 859, L20, doi: [10.3847/2041-8213/aab429](https://doi.org/10.3847/2041-8213/aab429)
- Dekany, R., Smith, R. M., Riddle, R., et al. 2020, *PASP*, 132, 038001, doi: [10.1088/1538-3873/ab4ca2](https://doi.org/10.1088/1538-3873/ab4ca2)
- Earl, N., French, K. D., Ramirez-Ruiz, E., et al. 2025, *ApJ*, 983, 28, doi: [10.3847/1538-4357/adb974](https://doi.org/10.3847/1538-4357/adb974)
- Eracleous, M. 1998, *Advances in Space Research*, 21, 33, doi: [10.1016/S0273-1177\(97\)00612-1](https://doi.org/10.1016/S0273-1177(97)00612-1)
- Eracleous, M., & Halpern, J. P. 2003, *ApJ*, 599, 886, doi: [10.1086/379540](https://doi.org/10.1086/379540)
- Eracleous, M., Livio, M., Halpern, J. P., & Storchi-Bergmann, T. 1995, *ApJ*, 438, 610, doi: [10.1086/175104](https://doi.org/10.1086/175104)
- Flohic, H. M. L. G., Eracleous, M., & Bogdanović, T. 2012, *ApJ*, 753, 133, doi: [10.1088/0004-637X/753/2/133](https://doi.org/10.1088/0004-637X/753/2/133)
- Gezari, S., Halpern, J. P., & Eracleous, M. 2007, *ApJS*, 169, 167, doi: [10.1086/511032](https://doi.org/10.1086/511032)
- Giustini, M., & Proga, D. 2019, *A&A*, 630, A94, doi: [10.1051/0004-6361/201833810](https://doi.org/10.1051/0004-6361/201833810)
- Graham, M. J., Kulkarni, S. R., Bellm, E. C., et al. 2019, *PASP*, 131, 078001, doi: [10.1088/1538-3873/ab006c](https://doi.org/10.1088/1538-3873/ab006c)
- GRAVITY Collaboration, Amorim, A., Bourdarot, G., et al. 2024, *A&A*, 684, A167, doi: [10.1051/0004-6361/202348167](https://doi.org/10.1051/0004-6361/202348167)
- Ho, L. C., Rudnick, G., Rix, H.-W., et al. 2000, *ApJ*, 541, 120, doi: [10.1086/309440](https://doi.org/10.1086/309440)
- Hoffman, M., Sountsov, P., Dillon, J. V., et al. 2019, arXiv e-prints, arXiv:1903.03704, doi: [10.48550/arXiv.1903.03704](https://doi.org/10.48550/arXiv.1903.03704)
- Holoien, T. W. S., Huber, M. E., Shappee, B. J., et al. 2019, *ApJ*, 880, 120, doi: [10.3847/1538-4357/ab2ae1](https://doi.org/10.3847/1538-4357/ab2ae1)
- Hopkins, B., & Skellam, J. G. 1954, *Annals of Botany*, 18, 213, doi: [10.1093/oxfordjournals.aob.a083391](https://doi.org/10.1093/oxfordjournals.aob.a083391)
- Huang, X., Davis, S. W., & Jiang, Y.-f. 2024, *ApJ*, 974, 165, doi: [10.3847/1538-4357/ad6c39](https://doi.org/10.3847/1538-4357/ad6c39)
- Hung, T., Foley, R. J., Ramirez-Ruiz, E., et al. 2020, *The Astrophysical Journal*, 903, 31, doi: [10.3847/1538-4357/abb606](https://doi.org/10.3847/1538-4357/abb606)
- Kollatschny, W., Reichstein, A., & Zetzl, M. 2012, *A&A*, 548, A37, doi: [10.1051/0004-6361/201118174](https://doi.org/10.1051/0004-6361/201118174)
- Kollatschny, W., & Zetzl, M. 2013, *A&A*, 549, A100, doi: [10.1051/0004-6361/201219411](https://doi.org/10.1051/0004-6361/201219411)
- Krolik, J., Piran, T., Svirski, G., & Cheng, R. M. 2016, *ApJ*, 827, 127, doi: [10.3847/0004-637X/827/2/127](https://doi.org/10.3847/0004-637X/827/2/127)
- Lewis, K. T., Eracleous, M., & Storchi-Bergmann, T. 2010, *ApJS*, 187, 416, doi: [10.1088/0067-0049/187/2/416](https://doi.org/10.1088/0067-0049/187/2/416)
- Liu, F. K., Zhou, Z. Q., Cao, R., Ho, L. C., & Komossa, S. 2017, *MNRAS*, 472, L99, doi: [10.1093/mnrasl/slx147](https://doi.org/10.1093/mnrasl/slx147)
- McInnes, L., Healy, J., & Melville, J. 2018, arXiv e-prints, arXiv:1802.03426, doi: [10.48550/arXiv.1802.03426](https://doi.org/10.48550/arXiv.1802.03426)
- McKinney, J. C., Dai, L., & Avara, M. J. 2015, *MNRAS*, 454, L6, doi: [10.1093/mnrasl/slv115](https://doi.org/10.1093/mnrasl/slv115)
- Metzger, B. D., & Stone, N. C. 2016, *MNRAS*, 461, 948, doi: [10.1093/mnras/stw1394](https://doi.org/10.1093/mnras/stw1394)
- Murray, N., & Chiang, J. 1996, *Nature*, 382, 789, doi: [10.1038/382789a0](https://doi.org/10.1038/382789a0)
- Negus, J., Comerford, J. M., & Sánchez, F. M. 2024, *ApJ*, 971, 92, doi: [10.3847/1538-4357/ad4c68](https://doi.org/10.3847/1538-4357/ad4c68)
- Ohsuga, K., & Mineshige, S. 2011, *ApJ*, 736, 2, doi: [10.1088/0004-637X/736/1/2](https://doi.org/10.1088/0004-637X/736/1/2)
- Pariev, V. I., Blackman, E. G., & Boldyrev, S. A. 2003, *A&A*, 407, 403, doi: [10.1051/0004-6361:20030868](https://doi.org/10.1051/0004-6361:20030868)
- Parkinson, E. J., Knigge, C., Matthews, J. H., et al. 2022, *MNRAS*, 510, 5426, doi: [10.1093/mnras/stac027](https://doi.org/10.1093/mnras/stac027)
- Phan, D., Pradhan, N., & Jankowiak, M. 2019, arXiv e-prints, arXiv:1912.11554, doi: [10.48550/arXiv.1912.11554](https://doi.org/10.48550/arXiv.1912.11554)
- Piran, T., Svirski, G., Krolik, J., Cheng, R. M., & Shiokawa, H. 2015, *ApJ*, 806, 164, doi: [10.1088/0004-637X/806/2/164](https://doi.org/10.1088/0004-637X/806/2/164)
- Popović, L. Č., Mediavilla, E., Bon, E., & Ilić, D. 2004, *A&A*, 423, 909, doi: [10.1051/0004-6361:20034431](https://doi.org/10.1051/0004-6361:20034431)
- Proga, D., Stone, J. M., & Kallman, T. R. 2000, *ApJ*, 543, 686, doi: [10.1086/317154](https://doi.org/10.1086/317154)
- Rees, M. J. 1988, *Nature*, 333, 523, doi: [10.1038/333523a0](https://doi.org/10.1038/333523a0)
- Schimoia, J. S., Storchi-Bergmann, T., Nemmen, R. S., Winge, C., & Eracleous, M. 2012, *ApJ*, 748, 145, doi: [10.1088/0004-637X/748/2/145](https://doi.org/10.1088/0004-637X/748/2/145)
- Schimoia, J. S., Storchi-Bergmann, T., Winge, C., Nemmen, R. S., & Eracleous, M. 2017, *MNRAS*, 472, 2170, doi: [10.1093/mnras/stx2107](https://doi.org/10.1093/mnras/stx2107)
- Shakura, N. I., & Sunyaev, R. A. 1973, *A&A*, 24, 337
- Shapovalova, A., Popović, L., Bochkarev, N., et al. 2009, *New Astronomy Reviews*, 53, 191, doi: [https://doi.org/10.1016/j.newar.2009.08.004](https://doi.org/https://doi.org/10.1016/j.newar.2009.08.004)
- Shiokawa, H., Krolik, J. H., Cheng, R. M., Piran, T., & Noble, S. C. 2015, *ApJ*, 804, 85, doi: [10.1088/0004-637X/804/2/85](https://doi.org/10.1088/0004-637X/804/2/85)
- Short, P., Nicholl, M., Lawrence, A., et al. 2020, *MNRAS*, 498, 4119, doi: [10.1093/mnras/staa2065](https://doi.org/10.1093/mnras/staa2065)
- Sądowski, A., Tejada, E., Gafton, E., Rosswog, S., & Abarca, D. 2016, *MNRAS*, 458, 4250, doi: [10.1093/mnras/stw589](https://doi.org/10.1093/mnras/stw589)
- Steinberg, E., & Stone, N. C. 2024, *Nature*, 625, 463, doi: [10.1038/s41586-023-06875-y](https://doi.org/10.1038/s41586-023-06875-y)

- Storchi-Bergmann, T., Eracleous, M., Teresa Ruiz, M., et al. 1997, *ApJ*, 489, 87, doi: [10.1086/304783](https://doi.org/10.1086/304783)
- Storchi-Bergmann, T., Nemmen da Silva, R., & Eracleous, M. 2003a, in *Astronomical Society of the Pacific Conference Series*, Vol. 297, *Star Formation Through Time*, ed. E. Perez, R. M. Gonzalez Delgado, & G. Tenorio-Tagle, 431, doi: [10.48550/arXiv.astro-ph/0211477](https://doi.org/10.48550/arXiv.astro-ph/0211477)
- Storchi-Bergmann, T., Schimoia, J. S., Peterson, B. M., et al. 2017, *ApJ*, 835, 236, doi: [10.3847/1538-4357/835/2/236](https://doi.org/10.3847/1538-4357/835/2/236)
- Storchi-Bergmann, T., Nemmen da Silva, R., Eracleous, M., et al. 2003b, *ApJ*, 598, 956, doi: [10.1086/378938](https://doi.org/10.1086/378938)
- Strateva, I. V., Strauss, M. A., Hao, L., et al. 2003, *AJ*, 126, 1720, doi: [10.1086/378367](https://doi.org/10.1086/378367)
- Strubbe, L. E., & Quataert, E. 2009, *MNRAS*, 400, 2070, doi: [10.1111/j.1365-2966.2009.15599.x](https://doi.org/10.1111/j.1365-2966.2009.15599.x)
- Strubbe, L. E., & Quataert, E. 2011, *MNRAS*, 415, 168, doi: [10.1111/j.1365-2966.2011.18686.x](https://doi.org/10.1111/j.1365-2966.2011.18686.x)
- Ward, C., Gezari, S., Nugent, P., et al. 2024, *ApJ*, 961, 172, doi: [10.3847/1538-4357/ad147d](https://doi.org/10.3847/1538-4357/ad147d)
- Wevers, T., Nicholl, M., Guolo, M., et al. 2022, *A&A*, 666, A6, doi: [10.1051/0004-6361/202142616](https://doi.org/10.1051/0004-6361/202142616)
- Yong, S. Y., Webster, R. L., King, A. L., et al. 2017, *PASA*, 34, e042, doi: [10.1017/pasa.2017.37](https://doi.org/10.1017/pasa.2017.37)
- Zhang, M., Zhang, W., Deng, H., Guo, H., & Sun, J. 2025, *ApJ*, 993, 244, doi: [10.3847/1538-4357/ae0581](https://doi.org/10.3847/1538-4357/ae0581)